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The Hermite Transform: An Efficient Tool for Noise Reduction and Image Fusion in Remote-Sensing

Boris Escalante-Ramírez and Alejandra A. López-Caloca

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24.1 Introduction

In this chapter, we introduce the Hermite transform as an efficient tool for remote-sensing image processing applications. The Hermite transform is an image representation model that mimics some of the more important properties of human visual perception, namely the local orientation analysis and the Gaussian derivative model of early vision. We limit our discussion to the cases of noise reduction and image fusion. However, many different applications can be tackled within the scheme of direct-inverse Hermite transform.

It is generally acknowledged that visual perception models must involve two major processing stages: (1) initial measurements and (2) high-level interpretation. Fleet and Jepson [1] pointed out that the early measurement is a rich encoding of image structure in terms of generic properties from which structures that are more complex are easily detected and analyzed. Such measurement processes should be image-independent and require no previous or concurrent interpretation. Unfortunately, it is not known what primitives are necessary and sufficient for interpretation or even identification of meaningful features. However, we know that, for image processing purposes, linear operators that exhibit special kind of symmetries related to translation, rotation, and magnification are of particular interest. A family of generic neighborhood operators fulfilling these
requirements is that formed by the so-called Gaussian derivatives [2]. These operators have long been used in computer vision for feature extraction [3,4], and are relevant in visual system modeling [5]. Formal integration of these operators is achieved in the Hermite transform introduced first by Martens [6,7], and recently reformulated as a multi-scale image representation model for local orientation analysis [8,9]. This transform can take many alternative forms corresponding to different ways of coding local orientations in the image.

Young showed that Gaussian derivatives model the measured receptive field data more accurately than the Gabor functions do [10]. Like the receptive fields, both Gabor functions and Gaussian derivatives are spatially local and consist of alternating excitatory and inhibitory regions within a decaying envelope. However, the Gaussian derivative analysis is found to be more efficient because it takes advantage of the fact that Gaussian derivatives comprise an orthogonal basis if they belong to the same point of analysis. Gaussian derivatives can be interpreted as local generic operators in a scale-space representation described by the isotropic diffusion equation [2]. In a related work, the Gaussian derivatives have been interpreted as the product of Hermite polynomials and a Gaussian window [6], where windowed images are decomposed into a set of Hermite polynomials. Some mathematical models based on these operators at a single spatial scale have been described elsewhere [6,11]. In the case of the Hermite Transform, it has been extended to the multi-scale case [7–9], and has been successfully used in different applications such as noise reduction [12], coding [13], and motion estimation for the case of image sequences [14].

Applications to local orientation analysis are a major concern in this chapter. It is well known that local orientation estimation can be achieved by combining the outputs from polar separable quadrature filters [15]. Freeman and Adelson developed a technique to steer filters by linearly combining basis filters oriented at a number of specific directions [16]. The possibilities are, in fact, infinite because the set of basis functions required to steer a function is not unique [17]. The Gaussian derivative family is perhaps the most common example of such functions.

In the first part of this chapter we introduce the Hermite transform as an image representation model, and show how local analysis can be achieved from a steered Hermite transform.

In the second part we build a noise-reduction algorithm for SAR images based on the steered Hermite transform that adapts to the local image content and to the multiplicative nature of speckle.

In the third section, we fuse multi-spectral and panchromatic images from the same satellite (Landsat ETM+) with different spatial resolutions. In this case we show how the proposed method improves spatial resolution and preserves the spectral characteristics, that is, the biophysical variable interpretation of the original images remains intact.

Finally, we fuse SAR and multi-spectral Landsat ETM+ images, and show that in this case spatial resolution is also improved while spectral resolution is preserved. Speckle reduction in the SAR image is achieved, along with image fusion, within the analysis-synthesis process of the fusion scheme.

Both fusion and speckle-reduction algorithms are based on the detection of relevant image structures (primitives) during the analysis stage. For this purpose, Gaussian-derivative filters at different scales can be used. Local orientation is estimated so that the transform can be rotated at every position of the analysis window. In the case of noise reduction, transform coefficients are classified based on structure dimensionality and energy content so that those belonging to speckle are discarded. With a similar criterion, transform coefficients from different image sources are classified to select coefficients from each image that contribute to synthesize the fused image.
24.2 The Hermite Transform

24.2.1 The Hermite Transform as an Image Representation Model

The Hermite transform \([6,7]\) is a special case of polynomial transform. It can be regarded as an image description model. Firstly, windowing with a local function \(v(x, y)\) takes place at several positions over the input image. Next, local information at every analysis window is expanded in terms of a family of orthogonal polynomials. The polynomials \(G_{m,n}(x, y)\) used to approximate the windowed information are determined by the analysis window function and satisfy the orthogonal condition:

\[
\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \omega^2(x, y)G_{m,n}(x, y)G_{l,k}(x, y)\, dx\, dy = \delta_{mk}\delta_{nl} \tag{24.1}
\]

for \(n, k = 0, \ldots, \infty; m = 0; n, l = 0, \ldots, k\); where \(\delta_{mk}\) denotes the Kronecker function.

Psychophysical insights suggest using a Gaussian window function, which resembles the receptive field profiles of human vision, that is,

\[
\omega(x, y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{(x^2 + y^2)}{2\sigma^2}\right) \tag{24.2}
\]

The Gaussian window is separable into Cartesian coordinates; it is isotropic, thus it is rotationally invariant and its derivatives are good models of some of the more important retinal and cortical cells of the human visual system \([5,10]\).

In the case of a Gaussian window function, the associated orthogonal polynomials are the Hermite polynomials \([18]\):

\[
G_{n-m,m}(x, y) = \frac{1}{\sqrt{2^n(n-m)!m!}} H_{n-m}(\frac{x}{\sigma})H_m(\frac{y}{\sigma})\tag{24.3}
\]

where \(H_n(x)\) denotes the \(n\)th Hermite polynomial.

The original signal \(L(x, y)\), where \((x, y)\) are the pixel coordinates, is multiplied by the window function \(\omega(x-p, y-q)\), at positions \(p, q\) that conform the sampling lattice \(S\).

Through replication of the window function over the sampling lattice, a periodic weighting function is defined as \(W(x,y) = \sum_{(p,q) \in S} \omega(x-p, y-q)\). This weighting function must be different from zero for all coordinates \((x, y)\), then:

\[
L(x, y) = \frac{1}{W(i,j)} \sum_{(p,q) \in S} L(x, y)\omega(x-p, y-q) \tag{24.4}
\]

The signal content within every window function is described as a weighted sum of polynomials \(G_{m,n-m}(x, y)\) of \(m\) degree in \(x\) and \(n - m\) in \(y\). In a discrete implementation, the Gaussian window function may be approximated by the binomial window function, and in this case, its orthogonal polynomials \(G_{m,n-m}(x, y)\) are known as the Krawtchouck’s polynomials.
In either case, the polynomial coefficients \( L_{m,n-m}(p, q) \) are calculated by convolution of the original image \( L(x, y) \) with the function filter \( D_{m,n-m}(x, y) = G_{m,n-m}(-x, -y) \omega^2(-x, -y) \) followed by subsampling at positions \((p, q)\) of the sampling lattice \( S \), i.e.,

\[
L_{m, n-m}(p,q) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} L(x, y) D_{m, n-m}(x - p, y - q) dx \, dy
\] (24.5)

For the case of the Hermite transform, it can be shown [18] that the filter functions \( D_{m,n-m}(x, y) \) correspond to Gaussians derivatives of order \( m \) in \( x \) and \( n - m \) in \( y \), in agreement with the Gaussian derivative model of early vision [5, 10].

The process of recovering the original image consists of interpolating the transform coefficients with the proper synthesis filters. This process is called an inverse polynomial transform and is defined by

\[
\hat{L}(x, y) = \sum_{n=0}^{\infty} \sum_{m=0}^{n} \sum_{(p, q) \in S} L_{m, n-m}(p, q) P_{m, n-m}(x-p, y-q)
\] (24.6)

The synthesis filters \( P_{m,n-m}(x, y) \) of order \( m \times (n - m) \), are defined by

\[
P_{m,n-m}(x, y) = \frac{G_{m, n-m}(x, y) \omega(x, y)}{W(x, y)} \quad \text{for} \quad m = 0, \ldots, n \text{ and } n = 0, \ldots, \infty.
\]

Figure 24.1 shows the analysis and synthesis stages of a polynomial transform. Figure 24.2 shows a Hermite transform calculated on a satellite image.

To define a polynomial transform, some parameters have to be chosen. First, we have to define the characteristics of the window function. The Gaussian window is the best option from a perceptual point of view and from the scale-space theory. Other free parameters are the size of the Gaussian window spread (\( \sigma \)) and the distance between adjacent window positions (sampling lattice). The size of the window functions must be related to the spatial scale of the image structures that are to be analyzed. Fine local changes are better detected with small windows, but on the contrary, representation of low-resolution objects needs large windows. To overcome this compromise, multi-resolution representations are a good alternative. For the case of the Hermite transform, a multi-resolution extension has recently been proposed [8, 9].

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**FIGURE 24.1**

Analysis and synthesis with the polynomial transform.
The Steered Hermite Transform

The Hermite transform has the advantage that high-energy compaction can be obtained through adaptively steering the transform [19]. The term steerable filters describes a set of filters that are rotated copies of each other, and a copy of the filter in any orientation which is then constructed as a linear combination of a set of basis filters. The steering property of the Hermite filters can be considered because the filters are products of polynomials with a radially symmetric window function. The $N+1$ Hermite filters of $N$th-order form a steerable basis for each individual filter of order $N$. Based on the steering property, the Hermite filters at each position in the image adapt to the local orientation content. This adaptability results in significant compaction.

For orientation analysis purposes, it is convenient to work with a rotational version of the Hermite transform. The polynomial coefficients can be computed through a convolution of the image with the filter functions $D_m(x)D_n(y)$; the properties of the filter functions are separable in spatial and polar domains and the Fourier transform of the filter functions are expressed in polar coordinates considering $\omega_x = \omega \cos \theta$ and $\omega_y = \omega \sin \theta$,

$$d_m(\omega_x)d_{n-m}(\omega_y) = g_{m,n-m}(\theta) \cdot d_n(\omega)$$ (24.7)

where $d_n(\omega)$ is the Fourier transform for each filter function, and the radial frequency of the filter function of the $n$th order Gaussian derivative is given by

$$d_n(\omega) = \frac{1}{\sqrt{2^n n!}} (-j\omega \sigma)^n \exp\left(-\left(\omega \sigma\right)^2/4\right)$$ (24.8)

and the orientation selectivity of the filter is expressed by

$$g_{m,n-m}(\theta) = \sqrt{\frac{n}{m}} \cos^m \theta \cdot \sin^{n-m} \theta$$ (24.9)
In terms of orientation frequency functions, this property of the Hermite filters can be expressed by

\[ g_{m,n}(\theta - \theta_0) = \sum_{k=0}^{n} c_{m,k}(\theta_0) g_{n-k,k}(\theta) \]  

(24.10)

where \( c_{m,k}(\theta_0) \) is the steering coefficient. The Hermite filter rotation at each position over the image is an adaptation to local orientation content. Figure 24.3 shows the directional Hermite decomposition over an image. First a Hermite transform was applied and then the coefficients of this transform were rotated towards the local estimated orientation, according to a maximum oriented energy criterion at each window position. For local 1D patterns, the steered Hermite transform provides a very efficient representation. This representation consists of a parameter \( \theta \), indicating the orientation of the pattern, and a small number of coefficients, representing the profile of the pattern perpendicular to its orientation. For a 1D pattern with orientation \( \theta \), the following relation holds:

\[
L^{\theta}_{n-m,m} = \begin{cases} 
\sum_{k=0}^{n} g_{n-k,k}(\theta) L_{n-k,k,m} & m = 0 \\
0, & m > 0
\end{cases} 
\]  

(24.11)

For such a pattern, steering over \( \theta \) results in a compaction of energy into the coefficients \( L^{\theta}_{n-0,m} \), while all other coefficients are set to zero.

The energy content can be expressed through the Hermite coefficients (Parseval Theorem) as

\[
E_\infty = \sum_{n=0}^{\infty} \sum_{m=0}^{n} [L_{n-m,m}]^2 
\]  

(24.12)

The energy up to order \( N \), \( E_N \) is defined as the addition of all squared coefficients up to \( N \) order.

**FIGURE 24.3**

Steered Hermite transform. Left: Original coefficients. Right: Steered coefficients. It can be noted that most coefficient energy is concentrated on the upper row.
The steered Hemite transform offers a way to describe 1D patterns on the basis of their orientation and profile. We can differentiate 1D energy terms and 2D energy terms. That is, for each local signal we have

\[ E_{N}^{1D}(\theta) = \sum_{n=1}^{N} \left[ L_{n,0}^{\theta} \right]^2, \]  
(24.13)

\[ E_{N}^{2D}(\theta) = \sum_{n=1}^{N} \sum_{m=1}^{n} \left[ L_{n-m,m}^{\theta} \right]^2, \]  
(24.14)

24.3 Noise Reduction in SAR IMAGES

The use of synthetic aperture radar (SAR) images instead of visible and multi-spectral images is becoming increasingly popular, because of their capability of imaging even in the case of cloud-covered remote areas. In addition to the all-weather capacity, there are several well-known advantages of SAR data over other imaging systems [20]. Unfortunately, the poor quality of SAR images makes it very difficult to perform direct information extraction tasks. Even more, the incorporation of external reference data (in-situ measurements) is frequently needed to guaranty a good positioning of the results. Numerous filters have been proposed to remove speckle in SAR imagery; however, in most cases and even in the most elegant approaches, filtering algorithms have a tendency to smooth speckle as well as information. For numerous applications, low-level processing of SAR images remains a partially unsolved problem. In this context, we propose a restoration algorithm that adaptively smoothes images. Its main advantage is that it retains subtle details.

The Hermite transform coefficients are used to discriminate noise from relevant information such as borders and lines in a SAR image. Then an energy mask containing relevant image locations is built by thresholding the first-order transform coefficient energy \( E_{1} \): 
\[ E_{1} = L_{0,1}^{2} + L_{1,0}^{2} \] where \( L_{0,1} \) and \( L_{1,0} \) are the first-order coefficients of the Hermite transform. These coefficients are obtained by convolving the original image with the first-order derivatives of a Gaussian function, which are known to be quasi-optimal edge detectors [21]; therefore, the first-order energy can be used to discriminate edges from noise by means of a threshold scheme.

The optimal threshold is set considering two important characteristics of SAR images. First, one-look amplitude SAR images have a Rayleigh distribution and the signal-to-noise ratio (SNR) is approximately 1.9131. Second, in general, the SNR of multi-look SAR images does not change over the whole image; furthermore, \( \text{SNR}_{\text{Nlooks}} = 1.9131 \sqrt{N} \), which yields for an homogeneous region \( l \):

\[ \sigma_l = \frac{\mu_l}{1.9131 \sqrt{N}} \]  
(24.15)

where \( \sigma_l \) is the standard deviation of the region \( l \), \( \mu_l \) is its mean value, and \( N \) is the number of looks of the image.

The first-order coefficient noise variance in homogeneous regions is given by

\[ \sigma^2 = \alpha \sigma_l^2, \]  
(24.16)
where
\[
\alpha = |R_L(x, y)^\top D_{1,0} o(x, y)^\top D_{1,0} o(-x, -y)|_{x=y=0}
\]

$R_L$ is the normalized autocorrelation function of the input noise, and $D_{1,0}$ is the filter used to calculate the first-order coefficient. Moreover, the probability density function (PDF) of $L_{1,0}$ and $L_{1,0}$ in uniform regions can be considered Gaussian, according to the Central Limit Theorem, then, the energy probability density function is exponential:
\[
P(E_1) = \frac{1}{2\sigma^2} \exp\left(-\frac{E_1}{2\sigma^2}\right) \quad (24.17)
\]

Finally, the threshold is fixed:
\[
T = 2 \ln\left(\frac{1}{P_R}\right) \sigma^2 \quad (24.18)
\]

where $P_R$ is the probability (percentage) of noise left on the image and will be set by the user. A careful analysis of this expression reveals that this threshold adapts to the local content of the image since Equation 24.15 and Equation 24.16 show the dependence of $\sigma$ on the local mean value $\mu$, the latter being approximated by the Hermite coefficient $L_{\infty}$.

With the locations of relevant edges detected, the next step is to represent these locations as one-dimensional patterns. This can be achieved by steering the Hermite transform as described in the previous section so that the steering angle $\theta$ is determined by the local edge orientation. Next, only coefficients $L_{n,0}^\theta$ are preserved, all others are set to zero.

In summary, the noise reduction strategy consists of classifying the image in either zero-dimensional patterns consisting of homogeneous noisy regions, or one-dimensional patterns containing noisy edges. The former are represented by the zeroth order Hermite transform, that is, the local mean value, and the latter by oriented 1D Hermite coefficients. When an inverse Hermite transform is performed over these selected coefficients, the resulting synthesized image consists of noise-free sharp edges and smoothed homogeneous regions. Therefore the denoised image preserves sharpness and thus, image quality. Some speckle remains in the image because there is always a compromise between the degree of noise reduction and the preservation of low-contrast edges. The user controls the balance of this compromise by changing the percentage of noise left $P_R$ on the image according to Equation 24.18.

Figure 24.4 shows the algorithm for noise reduction, and Figure 24.5 through Figure 24.8 show different results of the algorithm.

### 24.4 Fusion Based on the Hermite Transform

Image fusion has become a useful tool to enhance information provided by two or more sensors by combining the most relevant features of each image. A wide range of disciplines including remote sensing and medicine have taken advantage of fusion techniques, which in recent years have evolved from simple linear combinations to sophisticated methods based on principal components, color models, and signal transformations.
The Hermite Transform: An Efficient Tool for Noise Reduction

FIGURE 24.4
Noise-reduction algorithm.
among others [22–25]. Recently, multi-resolution techniques such as image pyramids and wavelet transforms have been successfully used [25–27]. Several authors have shown that, for image fusion, the wavelet transform approach offers good results [1,25,27]. Comparisons of Mallat’s and “à trous” methodologies have been studied [28]. Furthermore, multi-sensor image fusion algorithms based on intensity modulation have been proposed for SAR and multi-band optical data fusion [29].

Information in the fused image must lead to improved accuracy (from redundant information) and improved capacity (from complementary information). Moreover, from a visual perception point of view, patterns included in the fused image must be perceptually relevant and must not include distracting artifacts. Our approach aims at analyzing images by means of the Hermite transform, which allows us to identify perceptually relevant patterns to be included in the fusion process while discriminating spurious artifacts.

The steered Hermite transform has the advantage of energy compaction. Transform coefficients are selected with an energy compaction criterion from the steered Hermite
transform; therefore, it is possible to reconstruct an image with few coefficients and still preserve details such as edges and textures.

The general framework for fusion through Hermite transform includes five steps: (1) Hermite transform of the image. (2) Detection of maximum energy orientation with the energy measure $E_N^{1D}(\theta)$ at each window position. In practice, one estimator of the optimal orientation $\theta$ can be obtained through $\tan(\theta) = L_{0,1}/L_{1,0}$, where $L_{0,1}$ and $L_{1,0}$ are the first-order Hermite transform coefficients. (3) Adaptive steering of the transform coefficients, as described in previous sections. (4) Coefficient selection based on the method of verification of consistency [27]. This selection rule uses the maximum absolute value within a $5 \times 5$ window over the image (area of activity). The window variance is computed and used as a measurement of the activity associated with the central pixel of the window. In this way, a significant value indicates the presence of a dominant pattern.

FIGURE 24.7
Left: Original ERS1 image. Right: Image after noise reduction.

FIGURE 24.8
Left: Original ERS1 image. Right: Image after noise reduction.
in the local area. A map of binary decision is then created for the registry of the results. This binary map is subjected to consistency verification. (5) The final step of the fusion is the inverse transformation from the selected coefficients and their corresponding optimal $\theta^q$. Figure 24.9 shows a simplified diagram of this method.

### 24.4.1 Fusion Scheme with Multi-spectral and Panchromatic Images

Our objective of image fusion is to generate synthetic images with a higher resolution that attempts to preserve the radiometric characteristics of the original multi-spectral data. It is desirable that any procedure that fuses high-resolution panchromatic data with low-resolution multi-spectral data preserves, as much as possible, the original spectral characteristics.

To apply this image fusion method, it is necessary to resample the multi-spectral images so that their pixel size is the same as that of the panchromatic image’s. The steps for fusing multi-spectral and panchromatic images are as follows: (1) Generate new panchromatic images, whose histograms match those of each band of the multi-spectral image. (2) Apply the Hermite transform with local orientation extraction and detection of maximum energy orientation. (3) Select the coefficient based on the method of verification of consistency. (4) Inverse transformation with the optimal $\theta^q$ resulting from the selected coefficient set. This process of fusion is depicted in Figure 24.10.

**FIGURE 24.9** Fusion scheme with the Hermite transform.

**FIGURE 24.10** Hermite transform fusion for multi-spectral and panchromatic images.
24.4.2 Experimental Results

The proposed fusion scheme with multi-spectral images has been tested on optical data. We fused multi-spectral images from Landsat ETM+ (30 m) with its panchromatic band (15 m). We show in Figure 24.11 how the proposed method can help improve spatial resolution.

To evaluate the efficiency of the proposed method we calibrate the images so that digital values are transformed to reflectance values. Calibrated images were compared before and after fusion, by means of the Tasselep cap transformation (TCT) [30–32]. The TCT method is reported in [33]. The TCT transforms multi-spectral spatial values to a new domain based on biophysical variables, namely brightness, greenness, and a third component of the scene under study. It is deduced that the brightness component is a weighted sum of all the bands, based on the reflectance variation of the ground. The greenness component describes the contrast between near-infrared and visible bands with the mid-infrared bands. It is strongly related to the amount of green vegetation in the scene. Finally, the third component gives a measurement of the humidity content of the ground. Figure 24.12 shows the brightness, greenness, and third components obtained from Hermite-transform fusion results.

The TCT was applied to the original multi-spectral image, the Hermite transform (HT) fusion result, and principal component analysis (PCA) fusion method.

To understand the variability of TCT results on the original, HT fusion and PCA fusion images, the greenness, and brightness components were compared. The greenness and brightness components define the plane of vegetation in ETM+ data. These results are displayed in Figure 24.13. It can be noticed that, in the case of PCA, the brightness and greenness content differs considerably from the original image, while in the case of HT they are very similar to the original ones. A linear regression analysis of the TCT components (Table 24.1) shows that the brightness and greenness components of the HT-fused image present a high linear correlation with the original image values. In other words, the biophysical properties of multi-spectral images are preserved when using the HT for image fusion, in contrast to the case of PCA fusion.

![FIGURE 24.11](image)

(a) Original Landsat 7 ETM+ image of Mexico city (resampled to 15 m to match geocoded panchromatic), (b) Resulting image of ETM+ and panchromatic band fusion with Hermite transform (Gaussian window with spread $\sigma = 2$ and window spacing $d = 4$) (RGB composition 5–4–3).
24.4.3 Fusion Scheme with Multi-Spectral and SAR Images

In the case of SAR images, the characteristic noise, also known as speckle, imposes additional difficulties to the problem of image fusion. In spite of this limitation, the use of SAR images is becoming more popular due to their immunity to cloud coverage.

FIGURE 24.12
(a) Brightness, (b) greenness, and (c) third component in Hermite transform in a fused image.

FIGURE 24.13
Greenness versus brightness, (a) original multi-spectral, (b) HT fusion, (c) PCA fusion.
Speckle removal, as described in previous section, is therefore a mandatory task in fusion applications involving SAR imagery. The Hermite transform allows us to achieve both noise reduction and image fusion.

It is easy to figure out that local orientation analysis for the purpose of noise reduction can be combined with image fusion in a single direct-inverse Hermite transform scheme. Figure 24.14 shows the complete methodology to reduce noise and fuse Landsat ETM+ with SAR images.

24.4.4 Experimental Results

We fused multi-sensor images, namely SAR Radarsat (8 m) and multi-spectral Landsat ETM+ (30 m), with the Hermite transform and showed that in this case too spatial resolution was improved while spectral resolution was preserved. Speckle reduction in the SAR image was achieved, along with image fusion, within the analysis–synthesis process of the fusion scheme proposed. Figure 24.15 shows the result of panchromatic and SAR image HT fusion including speckle reduction. Figure 24.16 illustrates the result of multi-spectral and SAR image HT fusion. No significant distortion in the spectral and radiometric information is detected.

A comparison of the TCT of the original multi-spectral image and the fused image can be seen in Figure 24.17. There is a variation between both plots; however, the vegetation

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**TABLE 24.1**

Linear Regression Analysis of TCT Components: Correlation Factors of Original Image with HT Fusion and PCA Fusion Images

<table>
<thead>
<tr>
<th></th>
<th>Brightness (Original/HT)</th>
<th>Brightness (Original/PCA)</th>
<th>Greenness (Original/HT)</th>
<th>Greenness (Original/PCA)</th>
<th>Third Component (Original/HT)</th>
<th>Third Component (Original/PCA)</th>
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</thead>
<tbody>
<tr>
<td>Correlation factor</td>
<td>1.00</td>
<td>0.93</td>
<td>0.99</td>
<td>0.98</td>
<td>0.97</td>
<td>0.94</td>
</tr>
</tbody>
</table>

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**FIGURE 24.14**

Noise reduction and fusion for multi-spectral and SAR images.
FIGURE 24.15

FIGURE 24.16
(a) Original multi-spectral, (b) Result of ETM+ and Radarsat image fusion with HT (Gaussian window with spread $\sigma = 2$ and window spacing $d = 4$) (RGB composition 5–4–3).

FIGURE 24.17
Greenness versus brightness, (a) original multi-spectral, (b) LANDSAT–SAR fusion with HT.
plane remains similar, meaning that the fused image still can be used to interpret biophysical properties.

Another fusion result is displayed in Figure 24.18. In this case, the 5 m resolution SAR AeS-1 denoised image displayed on right side of Figure 24.5 is fused with its corresponding 25 m resolution LANDSAT TM5 image. Multi-spectral bands were analyzed with the principal components. The first component is shown on the left in Figure 24.18. Fusion of the latter with the SAR AeS-1 image is shown on the right. Note the resolution improvement of the fused image in comparison with the LANDSAT image.

24.5 Conclusions

In this chapter the Hermite transform was introduced as an efficient image representation model that can be used for noise reduction and fusion in remote perception imagery. Other applications such as coding and motion estimation have been demonstrated in related works [13,14].

In the case of noise reduction in SAR images, the adaptive algorithm presented here allows us to preserve image sharpness while smoothing homogeneous regions.

The proposed fusion algorithm based on the Hermite transform integrates images with different spatial and spectral resolutions, either from the same or different image sensors. The algorithm is intended to preserve both the highest spatial and spectral resolutions of the original data.

In the case of ETM+ multi-spectral and panchromatic image fusion, we demonstrated that the HT fusion method did not lose the radiometric properties of the original multi-spectral image; thus, the fused image preserved biophysical variable interpretation. Furthermore, the spatial resolution of the fused images was considerably improved.

In the case of SAR and ETM+ image fusion, spatial resolution of the fused image was also improved, and we showed for this case how noise reduction could be incorporated within the fusion scheme.

These algorithms present several common features, namely, detection of relevant image primitives, local orientation analysis, and Gaussian derivative operators, which are common to some of the more important characteristics of the early stages of human vision.
The algorithms presented here are formulated in a single spatial scale scheme, that is, the Gaussian window of analysis is fixed; however, multi-resolution is also an important characteristic of human vision and has also proved to be an efficient way to construct image processing solutions. Multi-resolution image processing algorithms are straightforward to build from the Hermite transform by means of hierarchical pyramidal structures that replicate, at each resolution level, the analysis–synthesis image processing schemes proposed here. Moreover, a formal approach to the multi-resolution Hermite transform for local orientation analysis has been recently developed, clearing the way to propose new multi-resolution image processing tasks [8,9].

Acknowledgment

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References


**Author Queries**

[AQ1] This sentence has been rephrased. Please advise whether the meaning has been retained.

[AQ2] Please check the limits of variables k, l, m, and n below the equation 24.1

[AQ3] Please note that the function sen is changed to sin (English equivalent, sine). Kindly approve this change.

[AQ4] Please check the number 9 given in the penultimate line in the paragraph before section 2.4.1. Is it part of theta?
[AQ5] Please advise whether the rephrasing is okay.

[AQ6] Please identify this heading uniquely within this section 4 (e.g. 4.2 Experimental Results with Panchromatic images / 4.4 Experimental Results with SAR images).

[AQ7] Please update this reference.