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Brief paper

Parameter tuning of second-order sliding mode controllers for linear plants with dynamic actuators☆

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Abstract

Tuning of second-order sliding mode control (2-SMC) algorithms in linear systems with dynamic actuators is considered. By means of a describing function (DF) approach, it is investigated how the parameters of a 2-SMC algorithm (the so-called "generalized sub-optimal" algorithm) affect the frequency and the magnitude of the limit cycles that occur when the overall relative degree of the plant plus the actuator is three or more. Explicit formulas are given that allow for setting the parameters of the algorithm to obtain a periodic solution with the prescribed characteristics. By means of simulation examples, we show that the estimated chattering parameters are in good agreement with the actual ones. We also show that the proposed design procedure can also be useful when the local linearization of a nonlinear dynamics is sufficiently accurate.

Keywords: Second-order sliding modes; Frequency analysis

1. Introduction

Sliding mode control (SMC) is a popular approach to control system design under heavy uncertainty, which remains one of the main subjects of modern control theory. SMC is precise, rather insensitive to disturbances (Utkin, 1992; Utkin, Guldner, & Shi, 1999), and usually very simple to implement.

The main drawbacks of classical first-order Sliding Modes (1-SM) are principally related to the so-called chattering effect (Boiko, 2003; Fridman, 2001; Utkin et al., 1999). The main cause of chattering has been identified as the presence of unmodelled parasitic dynamics in the switching devices (Bondarev, Bondarev, Kostylyeva, & Utkin, 1985).

Three main approaches to counteract the chattering phenomenon in SMC systems were proposed in the mid-eighties: the use of a continuous approximation of the relay (e.g. the saturation function (Burton & Zinober, 1986)), the use of an asymptotic state-observer to confine chattering in the observer dynamics bypassing the plant (Bondarev et al., 1985), the use of higher-order sliding mode control algorithms (Emelyanov, Korovin, & Levantovsky, 1986).

The higher-order sliding mode approach has been actively developed over the last two decades not only for chattering attenuation but also for the robust control of uncertain systems with relative degree two and higher (see Bartolini, Pisano, Punta, & Usai, 2003; Bartolini, Pisano, & Usai, 2001; Fridman & Levant, 2002; Levant, 1993, 2003; Orlov, Aguilar, & Cadiou, 2003; Sira-Ramirez, 2002 and references therein).

The main drawbacks of the continuous approximations and of the observed-based approach are the deterioration of accuracy and system robustness, respectively. In recent papers (Boiko & Fridman, 2004; Boiko, Fridman, & Castellanos, 2004; Boiko, Fridman, Pisano, & Usai, 2004a) it has been shown that even the second-order sliding mode (2-SM) algorithms suffer from

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chattering if parasitic dynamics are present increasing the system relative degree.

In this note we consider the generalized sub-optimal control algorithm (Bartolini et al., 2001, 2003), which is the generalization of a 2-SM control algorithm derived from the time-optimal control law of a double integrator (Bartolini, Ferrara, & Usai, 1997). We analyze the dependence of the frequency and the amplitude of chattering from the tuning parameters of the algorithm when it applied to dynamical systems with relative degree three and higher. The capability of affecting the frequency of the residual steady state oscillations may be useful, for example, to keep it far enough from the resonant frequencies of the plant.

There are two main approaches to chattering analysis: the time-domain analysis of the system dynamics or the use of frequency-domain techniques.

Analysis of the magnitude of the oscillations based on singularly perturbed relay systems was developed in Utkin (1992) and Fridman (2002). Poincare maps and LMIs have been suggested to investigate the existence and stability of periodic solutions in relay systems (Di Bernardo, Johansson, & Vasca, 2001; Goncalves, Megretski, & Dahleh, 2001 and the reference therein). A special decomposition of Poincare maps allowing for analyzing chattering in 1-SM control systems has been proposed in Fridman (2001). Preliminary results regarding the time-domain analysis of chattering in 2-SMC systems were presented in Boiko et al. (2004a).

When linear plants are considered several frequency-domain techniques can be used to assess the existence and stability of periodic solutions. The Tsypkin locus method (Tsypkin, 1984) provides exact values of the amplitude and frequency of chattering. The recently proposed "Locus of a Perturbed Relay System" (LPRS) method (Boiko, 2005) gives the exact values of chattering frequency and amplitudes and also allows to perform some robustness analysis (Boiko, 2003). All these approaches require quite tedious computations. Therefore the application of the approximate analysis method based on the Describing Function (DF) technique could be useful whenever the low-pass filtering condition is satisfied (Atherton, 1975). The DF method has already been used to estimate the frequency and the amplitude of the periodic motions in the 1-SMC systems (Shtessel & Lee, 1996; Zhilcov, 1974). The results obtained via the use of exact frequency-domain techniques feature a satisfactory correspondence with those obtained via the approximate DF method (Boiko, 2003).

In the present paper a parametric relay representation of the generalized sub-optimal 2-SMC algorithm (Bartolini et al., 2003) is given. The "Twisting" algorithm (Levant, 1993), the "sub-optimal" algorithm (Bartolini et al., 1997) and even the classical relay (Utkin, 1992) can be obtained as particular cases. Such representation is exploited for analysis and design purposes in the frequency domain in order to provide effective tuning rules for chattering attenuation. We assume that the cascade actuator-plant dynamics is a low-pass filter. We also assume that the steady-state oscillations are periodic, symmetric and with zero mean. The analysis results give the designer useful tuning rules to set the controller parameters so as the chattering effect is counteracted.

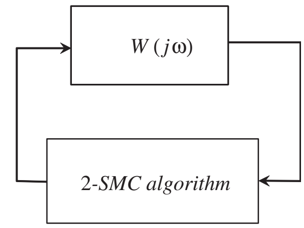

Fig. 1. The feedback control system.

This paper is organized as follows: in Section 2 we formulate the problem under investigation and detail the class of controlled plants we are concerned with. Section 3 contains the main results, namely a DF analysis of the considered class of control systems and the derivation of tuning rules for setting the parameters of the 2-SMC algorithm. In Section 4 the proposed tuning procedure is applied and verified by means of computer simulations. The estimated chattering parameters (amplitude and frequency of the periodic solution) obtained via the DF analysis are shown to be in good agreement with the simulated system's behaviour. Section 5 provides concluding remarks.

2. Problem formulation

Consider the feedback control system in Fig. 1.

Let the linear plant, including the actuator, be described by the state-space representation

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{b}u(t), & \mathbf{x} \in R^n, \quad u \in R, \\ y(t) = \mathbf{c}\mathbf{x}(t), & y \in R, \end{cases} \tag{1}$$

where $\mathbf{x}$ is the state vector, u is the actuator's input and $\mathbf{A}$, $\mathbf{b}$, $\mathbf{c}$ are matrices of appropriate dimensions. y can represent either the sliding variable or the plant output. Assume that matrix $\mathbf{A}$ is Hurwitz. The harmonic response $W(\mathrm{j}\omega) = \mathbf{c}(\mathrm{j}\omega\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}$ can be then used as the SISO plant model.

The 2-SMC algorithm is the generalized sub-optimal controller (Bartolini et al., 2003), i.e.

$$u(t) = -\alpha(t)U_M \operatorname{sign}(y(t) - \beta y_M(t)),$$

$$\alpha(t) = \begin{cases} 1 & \text{if } y_M(t)(y(t) - \beta y_M(t)) \geqslant 0, \\ \alpha^* & \text{if } y_M(t)(y(t) - \beta y_M(t)) < 0, \end{cases} \tag{2}$$

where $y_M(t)$ is a piece-wise constant function representing the value of the last *singular point* of $y(t)$ (i.e., the most recent value $y(t_{M_i})$ such that $\dot{y}(t_{M_i}) = 0$). U_M is the control authority, $\beta \in [0, 1)$ is the *anticipation* parameter and $\alpha^* \geqslant 1$ is the *modulation* parameter.

If the plant, even nonlinear, has relative degree two and its uncertain input–output dynamics can be represented as follows:

$$\ddot{y}(t) = \varphi(y(t), \dot{y}(t), t) + \gamma(y(t), \dot{y}(t), t)u(t),$$

$$|\varphi(y(t), \dot{y}(t), t)| \leqslant \Phi,$$

$$0 < \Gamma_m \leqslant \gamma(y(t), \dot{y}(t), t) \leqslant \Gamma_M, \tag{3}$$

then the control law (2) with the tuning conditions

$$U_M > \frac{\Phi}{\Gamma_m},$$

$$\alpha^* \in [1; +\infty) \cap \left(\frac{2\Phi + (1-\beta)\Gamma_M U_M}{(1+\beta)\Gamma_m U_M}; +\infty \right), \tag{4}$$

enforces a 2-SM onto the sliding surface $y=0$ (Bartolini et al., 2003). Special tuning of the controller (2) gives the "Twisting" algorithm (Levant, 1993) ($\beta=0$), the "sub-optimal" algorithm (Bartolini et al., 1997) ($\beta=0.5$) and the relay controller ($\beta=0, \alpha^*=1$).

The presence of dynamic actuators increasing the relative degree calls for special investigation techniques.

In this paper we show that *periodic* motions may occur in the linear case when the overall relative degree is three or more. We define a procedure, based on the DF method, devoted to estimate the frequency and maximal deviation from zero of the periodic solution. We also derive constructive controller tuning rules that allow to "shape" within a certain range the parameters (frequency and amplitude) of the periodic solution.

Remark 1. The combined effect of unmodelled stable linear actuators and matched disturbances has been analyzed in (Boiko, Fridman, Pisano, & Usai, 2004b). System (3) was augmented as follows:

$$\mu^m u^{(m)} + \sum_{i=0}^{m-1} p_i \mu^i u^{(i)} = v, \quad \mu > 0,$$

$$v = -\alpha(t) V_M \operatorname{sign}(y - \beta y_M), \tag{5}$$

where $u^{(0)} = u$ and coefficients p_i $(i = 0, \ldots, m-1)$ are such that the polynomial $P(s) = s^m + \sum_{i=0}^{m-1} p_i s^i$ is an Hurwitz one. It has been shown that bounded oscillations around the sliding manifold will eventually occur. It has also been shown that the amplitude of such oscillations is of order $O(\mu^2)$ and that there is no obvious way to affect the oscillation amplitude and frequency by changing the controller parameters. Tools developed in (Boiko, 2005; Fridman, 2001) could be promising to this end.

3. Describing-function analysis of the generalized sub-optimal algorithm

The generalized sub-optimal controller can be described as an active hysteretic relay whose hysteresis magnitude varies according to the past history of the relay input. The relay representation of controller (2) is given in Fig. 2.

Let us assume that the actual steady-state behaviour of the system (1), (2) is a periodic, unimodal symmetric motion with zero mean. Let y_M^p be the fixed point of the Poincare map $y(t_{M_{i+1}}) = -y(t_{M_i})$ (the construction of the Poincare map has been dealt with in Boiko et al., 2004a).

Then, the sequence of the sliding variable singular points, $y_{M_k} = y(t_{M_k})$ $(k=1,2,\ldots)$, is the alternating (ringing) sequence of positive and negative values $y_M^p, -y_M^p, y_M^p, -y_M^p$ (here "p" stands for periodic). The control sign switching would occur at the times when the system output is equal to βy_M^p.

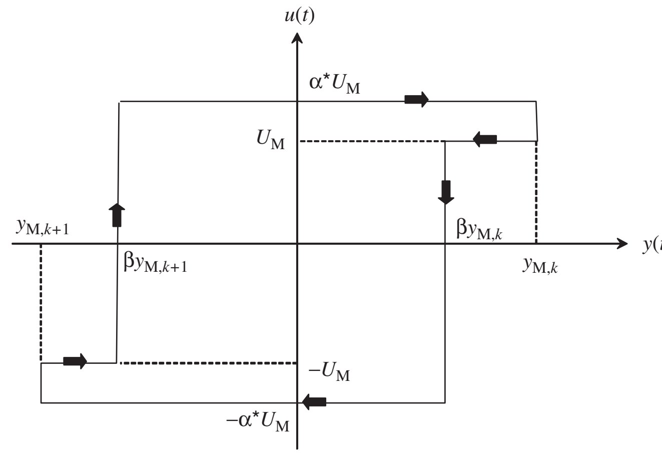

Fig. 2. Relay representation of the generalized sub-optimal controller.

The above assumptions regarding the characteristics of the periodic motion imply that in steady state the control u can be represented as a symmetric relay. Frequency domain methods (Atherton, 1975; Tsypkin, 1984) can therefore be used for the steady state motion analysis if the plant is a low-pass filter. In some cases asymmetric limit cycles can take place even with symmetric system nonlinearities (Di Bernardo et al., 2001). However, in this paper we will consider the symmetric case only.

The main difference between the analysis being done and the analysis of a conventional relay system is that the hysteresis value depends on y_M^p which is unknown. The problem can be solved by exploiting the fact that y_M^p represents the last singular point of y. The singular points of the output in a periodic process define the oscillation amplitude. Therefore, y_M^p is the unknown amplitude of the periodic solution.

The DF of the active hysteretic relay in Fig. 2, with $y_{M,k+1} = -y_{M,k} = -y_M^p$, is as follows (Atherton, 1975):

$$q(y_M^p) = \frac{2U_M}{\pi y_M^p}\left[(\alpha^*+1)\sqrt{1-\beta^2} + j[(\alpha^*-1)+\beta(\alpha^*+1)]\right]. \tag{6}$$

A periodic solution can appear if the negative reciprocal of the DF (6) intersects the Nyquist plot of the harmonic response $W(\mathrm{j}\omega)$ (Atherton, 1975). The negative reciprocal of the DF (6) is:

$$-\frac{1}{q} = \frac{\pi y_M^p}{4U_M} \frac{-(\alpha^*+1)\sqrt{1-\beta^2}}{\alpha^{*2}(1+\beta)+(1-\beta)} + \frac{\pi y_M^p}{4U_M} \frac{j[(\alpha^*-1)+\beta(\alpha^*+1)]}{\alpha^{*2}(1+\beta)+(1-\beta)}. \tag{7}$$

The locus (7) is a straight line backing out of the origin and forming a clockwise angle ψ with respect to the negative real axis (see Fig. 3).

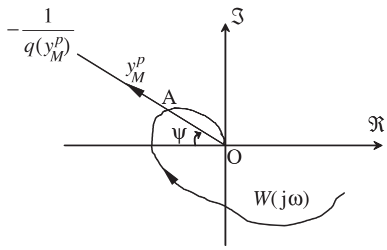

Fig. 3. DF-analysis in the complex plane.

From (7), angle ψ can be expressed as a function of the controller parameters

$$\psi = \arctan\left(\frac{(\alpha^*-1)+\beta(\alpha^*+1)}{(\alpha^*+1)\sqrt{1-\beta^2}}\right). \tag{8}$$

Note that ψ is ranging between 0 and $\pi/2$. Then, the periodic motion may exist if at some frequency $\omega=\overline{\omega}$ the phase characteristic of the actuator-plant transfer function ranges between $-\pi$ and $-\frac{3}{2}\pi$. This is only possible if the overall relative degree of the actuator/process system is higher than two. If an intersection point exists then the frequency of the periodic solution is $\overline{\omega}$ and the amplitude depends on the magnitude of $W(\mathrm{j}\omega)$ at the frequency $\overline{\omega}$. Usually, the larger ψ the greater the corresponding frequency $\overline{\omega}$, and the smaller the magnitude of the oscillation.

The magnitude of the steady-state oscillations y_M^p can be evaluated according to the following formula

$$M = |W(\mathrm{j}\overline{\omega})| = \frac{\pi y_M^p}{2\sqrt{2}U_M\sqrt{\alpha^{*2}(1+\beta)+(1-\beta)}}. \tag{9}$$

Direct use for design of the nonlinear formulas (8) and (9) can be avoided. In order to impose some prescribed constraints on the amplitude and frequency of the steady-state periodic oscillation it is convenient to refer to the curves in Fig. 4. A three-step tuning procedure can be given as follows:

A. Let $\omega_1 \leqslant \overline{\omega} \leqslant \omega_2$ be the desired range for the periodic solution frequency.
B. Evaluate $\psi_1 = -180° - \angle W(\mathrm{j}\omega_1)$ and $\psi_2 = -180° - \angle W(\mathrm{j}\omega_2)$.
C. Identify in the table reported in Fig. 4 proper values for the parameters α^* and β such that $\psi_1 \leqslant \psi \leqslant \psi_2$ while maximizing MU_M/y_M^p.

Maximizing MU_M/y_M^p, i.e. minimizing the oscillation amplitude y_M^p, is achieved by increasing β. However, it has been shown in Bartolini et al. (2003) that if β is too close to the unit value then the transient could slow down heavily. On-line parameter adjustment could be necessary, in some cases, in order to achieve satisfactory performance both during the transient and in steady-state.

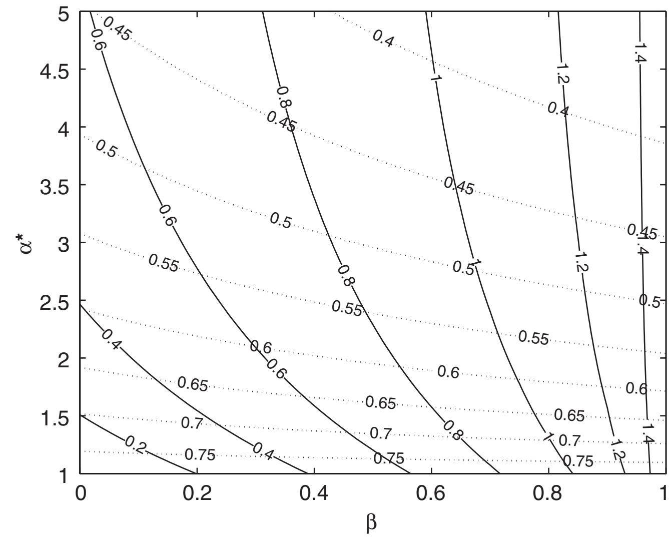

Fig. 4. Level sets of ψ (continuous lines) and MU_M/y_M^p (dotted lines) in the $\alpha^*-\beta$ plane.

	α^*	1		3	
β		**DF**	**Simulation**	**DF**	**Simulation**
0.5	$\overline{\omega}$	59.81	55.16	117.39	112.40
	y_M^p	$3.06\cdot10^{-3}$	$3.60\cdot10^{-3}$	$1.59\cdot10^{-3}$	$2.00\cdot10^{-3}$
0.8	$\overline{\omega}$	135.87	126.68	219.46	189.25
	y_M^p	$4.09\cdot10^{-4}$	$5.80\cdot10^{-4}$	$3.14\cdot10^{-4}$	$5.00\cdot10^{-4}$

Fig. 5. DF analysis vs. simulation results.

4. Simulations

4.1. An academic example

The plant-plus-actuator transfer function

$$W(s) = \frac{1}{(s^2+s+1)(1+0.01s)} \tag{10}$$

was considered for the closed-loop analysis with the generalized sub-optimal algorithm. The relative degree of the transfer function is three, then its Nyquist plot intersects the straight line (7).

Let us apply the described three-step procedure to shape the periodic solution parameters. Step A: the desired frequency range is $55\,\mathrm{rad\,s^{-1}} \leqslant \overline{\omega} \leqslant 65\,\mathrm{rad\,s^{-1}}$. Step B yields $\psi_1 = 0.49$ rad and $\psi_2 = 0.56$ rad. Interpolation of the curves in Fig. 4 is required. It follows that choosing $\alpha^*=1$ and $\beta=0.5$ one keeps ψ within the prescribed interval minimizing at the same time the amplitude of the periodic solution.

The theoretical (i.e. predicted by DF method) and actual (i.e. arising from the plant simulation) parameters of the periodic solution were compared also for different values of β and α^* (see the table in Fig. 5). The actual frequency of the periodic

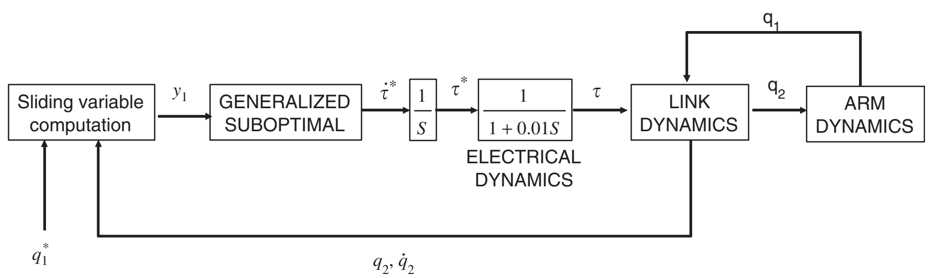

Fig. 6. Block scheme of the physical simulation example.

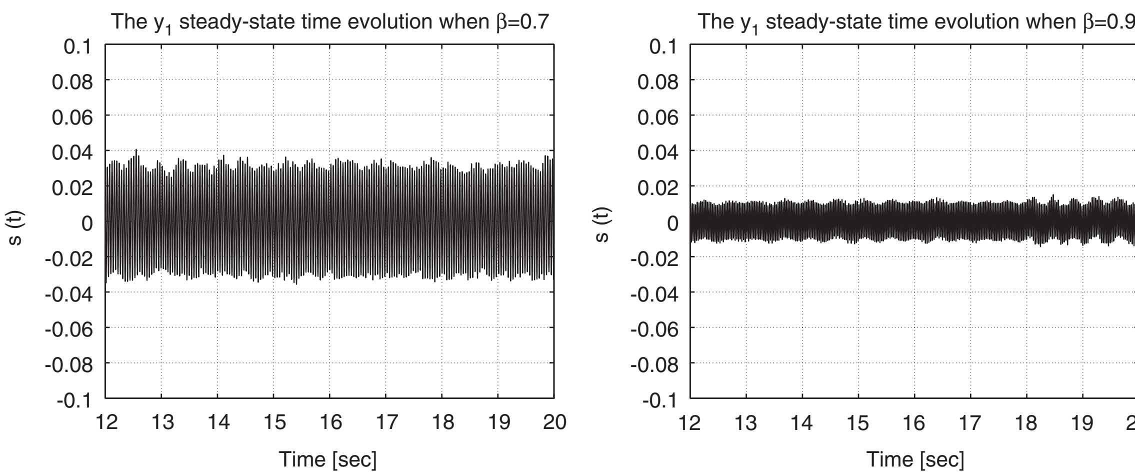

Fig. 7. The time evolution of y_1 when $\beta = 0.7$ (left) and $\beta = 0.9$ (right).

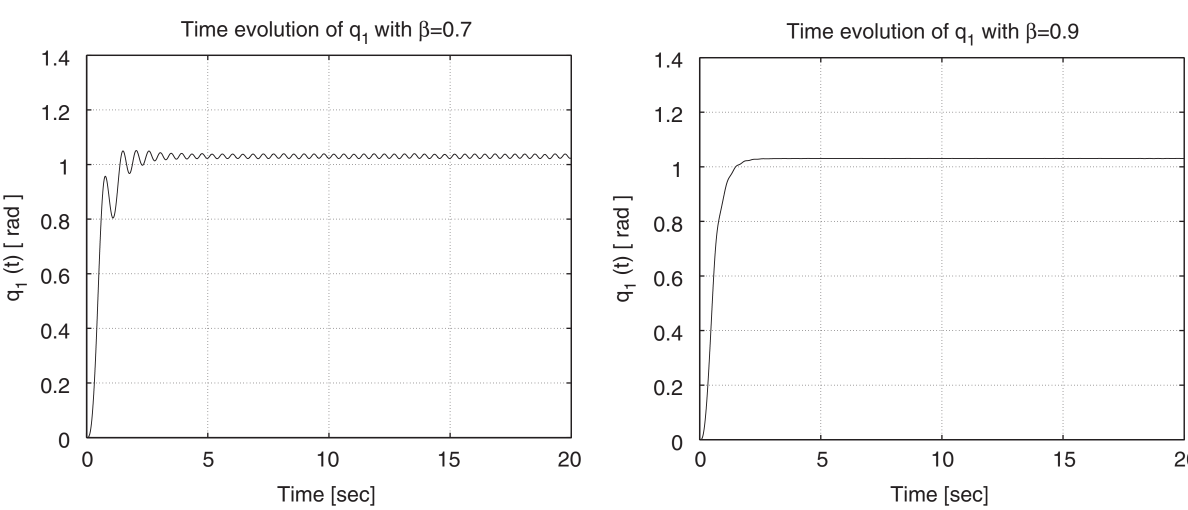

Fig. 8. The time evolution of q_1 when $\beta = 0.7$ (left) and $\beta = 0.9$ (right).

solution when $\alpha^* = 1$ and $\beta = 0.5$ lies within the prescribed interval.

4.2. A physical example

We are going to investigate, by means of a simulation example, if the proposed results are still applicable in the nonlinear setting. We argue that if the nonlinear system is steered near some constant operating point, and the corresponding local linearization is asymptotically stable, then the given tuning guidelines still apply. Consider the simplified model of a rotating arm driven by a torque motor through an elastic link

$$\tfrac{1}{2}ML^2\ddot{q}_1 + B\dot{q}_1 + \tfrac{1}{2}MgL\sin(q_1) = K(q_2 - q_1),$$

$$J\ddot{q}_2 + K(q_2 - q_1) = \tau, \tag{11}$$

where q_1 and q_2 represent the arm and motor coordinates, M and L are the mass and length of the arm, b is the viscous friction coefficient, J is the motor inertia reflected to the link-side of the gears and K is the joint stiffness coefficient. We consider the electrical dynamics of the drive as a first-order stable transfer function relating the "command" torque τ^* (output by the controller) and the actual torque acting on the link:

$$\tau = \frac{1}{1+\mu s}\tau^*, \quad \mu = 0.01. \tag{12}$$

The overall control system is represented in Fig. 6.

The task is to regulate the arm position to the constant set-point q_1^* by using only motor-side measurements (motor position and velocity).

The sliding variable is defined as $y_1 = \dot{q}_2 + c(q_2 - q_1^*)$, with c being a positive design parameter. Add an integrator at the input side ("anti-chattering 2-SMC design (Bartolini et al., 2003)) and set the discontinuous derivative of the command torque according to the generalized sub-optimal controller with $\alpha^* = 1$ and $U_M = 50$. The simulation parameters are: $M = 2$ Kg, $L = 1$ m, $J = 0.1\,\mathrm{Kg\,m^2}$, $b = 1$ Nm s, $K = 500$ Nm, $q_1^* = \pi/3$ rad, $c = 3$.

The relative degree between y_1 and $\dot{\tau}^*$ is three and the linearized dynamics around the target point is stable. Thus we can argue that in spite of the nonlinear dynamics of the system the sliding quantity y_1 could exhibit a periodic oscillation in steady-state.

The transfer function $G(s)$ between the arm coordinate deviation $\delta q_1 = q_1 - q_1^*$ and the sliding variable deviation $\delta y_1 = y_1$ is

$$G(s) = \frac{K}{(s+c)(\frac{1}{2}ML^2 s^2 + bs + \frac{1}{2}MgL\cos(q_1^*) + K)}. \tag{13}$$

The frequency response $G(j\omega)$ features a resonant mode near $25\,\mathrm{rad\,s^{-1}}$. In order to obtain at least -60 db of attenuation near the resonant frequency, the oscillation frequency of y_1 should be larger than $80\,\mathrm{rad\,s^{-1}}$. By considering the linearized transfer function between y and $\dot{\tau}^*$ it follows that angle ψ must be larger than 0.9 rad. In order to minimize at the same time the oscillation amplitude, the parameters β and α^* must be set according to $\beta > 0.8$, $\alpha^* \approx 1$.

The y_1 waveforms obtained when β=0.7 and 0.9 are reported in Fig. 7. As expected, increasing β the amplitude of the periodic oscillation reduces and the corresponding frequency increases. According to the previous considerations the behaviour of q_1 considerably improves by increasing β, as can be verified by comparing the plots in Fig. 8.

5. Conclusions

The describing function approach to the analysis of feedback control systems with linear plants driven by 2-SMC schemes has been presented. As a unified representation of several existing SMC algorithms, the generalized sub-optimal 2-SM controller is considered. The proposed graphical analysis indicates that if the plant-plus-actuator relative degree is three or more then a periodic solution can take place in steady-state. It has also been shown that changing the controller parameters allows varying the magnitude and frequency of the possible periodic solution. A constructive three-step procedure for shaping the periodic solution parameters via controller tuning has been developed and tested by simulations. The proposed approach proved to be effective to achieve a sufficiently accurate estimation of the steady-state control system performance.

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