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Variable structure control of synchronous generator: singularly perturbed analysis

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Synchronous generators have a natural different time scale dynamics. That is why, for modelling and control design in such systems, the methods of singular perturbations are widely used. In this paper the possibilities of sliding mode control design for synchronous generators are analysed. With this aim the concept of singular perturbation is revised in order to use it for relay control systems. The obtained results are used for sliding mode control of synchronous generator.

1. Introduction

The synchronous generators have a natural different time scale dynamics. That is why for modelling and control design in such systems the methods of singular perturbations are widely used. In this paper the possibilities of sliding mode control design for synchronous generators are analysed. With this aim the concept of singular perturbation is revised in order to use it for relay control systems. The obtained results are used for sliding mode control of synchronous generators.

Simplifications of plant models is a classical tool for electric power systems control design, and the most typical way to obtain reduced models is the use of the singular perturbation approach (Krause 1986, Kokotovic *et al*. 1986, Sauer and Kokotovic 1998, Sauer and Pai 1998). On the other hand, a fruitful and relatively simple approach, especially when dealing with non-linear plants subjected to perturbations, is based on variable structure control technique with sliding mode (Utkin 1977, 1992). First and foremost, this enables high accuracy and robustness to disturbances and plant parameter variations to be obtained. Second, the control variables of the basic sliding mode control law rapidly switch between extreme limits, that is ideal for the direct operation of the switched mode power converters of synchronous generators. However, applying discontinuous (relay) control to a plant model with the singular perturbation leads to some problems. Classical methods of singular perturbation (Kokotovic *et al*. 1986, Vasil'eva *et al*. 1995) are based on the spectrum separation and consequently these approaches need the smoothness of the models and control low. That is why the classical methods of singular perturbations are not valid for singularly perturbed relay control systems (SPRCS).

The decomposition methods for SPRCS were widely developed in the last fifteen years. Heck and Haddad (1989a, b, 2001) and Heck (1991) justified the motions separation and composite control for linear SPRCS with the first order sliding modes. Su (1999) decomposed SPRCS in two reduced subsystems by means of motion separation and a smooth approximation of a

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relay control. Some control algorithms for SPRCS were also developed in Innocenti *et al.* (2003). Fridman (2001, 2002b) has dedicated the decomposition techniques for the SPRCS with higher order sliding modes. In the works of Fridman (2002a, c) the different classes of periodic solutions were analysed.

The present paper discusses the advantages and possibilities of sliding mode control design for a special class of SPRCS describing synchronous generator dynamics. For the synchronous generators it is natural to use a two step control design (TSCD) procedure:

1. Eliminate the stator fast dynamics via singular perturbation methods and derive the reduced (6th order) model describing the slow mechanical and rotor flux dynamics.
2. Design a sliding mode excitation control low using block control technique (Loukianov 1998).

So the order of the original SPRCS is reduced in two steps: first the elimination of the fast dynamics and then the reduction of the slow dynamics model order into the sliding domain.

To justify the proposed TSCD procedure, first it is proved that the fast dynamics do not affect the entrance point into the sliding domain and the sliding mode equation of the slow dynamics outside of a boundary layer. Hence, the motion into the boundary layer does not affect the control design process. Then conditions of the uniform asymptotic stability for the original SPRCS, are found. The obtained results are used to design a sliding mode control low for synchronous generator angular speed and voltage.

This paper is organized as follows. Section 2 introduces the basic equations of the synchronous generator. In section 3 the concepts of singularly perturbed models with relay control are justified. In section 4 the singular perturbation approach is applied to design a synchronous generator controller. Simulations results illustrating the effectiveness of the proposed control are shown in section 5.

2. Synchronous generator models

2.1. *Basic equations*

The mathematical models for the synchronous generator are based on the mechanical and electric equilibrium equations (see for example Krause 1986). In electric power system analysis it is customary to use the per unit (p.u.) analysis that normalizes system variables, reduces computational effort and gives more clarity in the behaviour of the plant, Rankin (1945). The mechanical equilibrium equations for a synchronous generator are given by

$$\frac{d\delta}{dt} = \omega - \omega_b \tag{1}$$

$$\frac{d\omega}{dt} = \frac{\omega_b}{2H}(T_m - T_e), \tag{2}$$

where δ is the power angle (rad), ω is the angular velocity (rad s^{-1}), ω_b is the synchronous angular velocity (rad s^{-1}), H is the inertia constant (s), T_m is the mechanical torque (p.u.), and T_e is the electromechanical torque (p.u.). On the other hand, the electric equilibrium equations affected by the Park transformations (Park 1929), are expressed as

$$V = Ri + \omega G\varphi + \frac{d\varphi}{d\bar{t}} \tag{3}$$

$$\varphi = Li \tag{4}$$

where $\bar{t} = \omega_b t$, ω_b is the base angular velocity, t is the time in seconds, $\bar{t}$ is the time in p.u.,

$$V = \left[V_d, V_q, V_f, 0, 0, 0\right]^T \quad \varphi = \left(\varphi_d, \varphi_q, \varphi_f, \varphi_g, \varphi_{kd}, \varphi_{kq}\right)^T$$
$$i = \left(i_d, i_q, i_f, i_g, i_{kd}, i_{kq}\right)^T,$$

$$R = \begin{bmatrix} -r_s & & & & & \\ & -r_s & & & 0 & \\ & & r_f & & & \\ & & & r_g & & \\ & 0 & & & r_{kd} & \\ & & & & & r_{kq} \end{bmatrix},$$

$$L = \begin{bmatrix} -L_d & 0 & L_{md} & 0 & L_{md} & 0 \\ 0 & -L_q & 0 & L_{mq} & 0 & L_{mq} \\ -L_{md} & 0 & L_f & 0 & L_{md} & 0 \\ 0 & -L_{mq} & 0 & L_g & 0 & L_{mq} \\ -L_{md} & 0 & L_{md} & 0 & L_{kd} & 0 \\ 0 & -L_{mq} & 0 & L_{mq} & 0 & L_{kq} \end{bmatrix},$$

$$G = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

V_d and V_q are, respectively, the direct and quadrature stator voltages; V_f is the field excitation voltage; i_d and i_q are, respectively, the direct and quadrature stator currents; i_f is the field current; i_{kd}, i_g and i_{kq} are, respectively, the direct and quadrature axis damper winding currents; ϕ_d and ϕ_q are, respectively, the direct and quadrature stator fluxes; ϕ_f is the field flux; ϕ_{kd}, ϕ_g and ϕ_{kq} are, respectively, the direct and quadrature damper winding fluxes; r_s is the stator resistance; r_f is the field resistance; r_{kd}, r_g and r_{kq} are, respectively, the direct and quadrature damper winding resistances; L_d and L_q are, respectively, the direct and quadrature self inductances; L_{md} and L_{mq} are, respectively, the direct and quadrature magnetizing inductances; L_f is the field self inductance; L_{kd}, L_g and L_{kq} are, respectively, the direct and quadrature damper winding self inductances.

The equation for the electromechanical torque in terms of the currents and fluxes, is governed by

$$T_e = \varphi_d i_q - \varphi_q i_d. \tag{5}$$

The outside generator network equation expressed in p.u., is

$$V_{\text{gen}} = \frac{d\varphi_s}{dt} + G_{\text{ext}} i_s + V^{\infty}\gamma(\delta) \tag{6}$$

where

$$G_{\text{ext}} = \begin{bmatrix} R_{\text{ext}} & -\omega L_{\text{ext}} \\ \omega L_{\text{ext}} & R_{\text{ext}} \end{bmatrix}, \quad \gamma(\delta) = \begin{bmatrix} \sin\delta \\ \cos\delta \end{bmatrix},$$
$$V_{\text{gen}} = \begin{bmatrix} V_d \\ V_q \end{bmatrix}, \quad \varphi_s = \begin{bmatrix} \varphi_d \\ \varphi_q \end{bmatrix},$$

L_{ext} and R_{ext} are the transformer plus transmission line resistance and inductance, respectively; V^{∞} is the infinite bus voltage set at $1\angle 0$ (see Krause 1986).

2.2. *Time scale modelling*

To simplify the system model we will transform the system to the singularly perturbed form. With this aim it is necessary to find an adequate "parasite" parameter multiplying the stator dynamics. For that reason we scale the synchronous machine equations using the per unit representation.

2.2.1. Per unit rotor basic equations. The electric equilibrium equation for the field flux linkage in physical values is governed by

$$\bar{V}_f = \bar{r}_f \bar{i}_f + \frac{d\bar{\varphi}_f}{dt} \tag{7}$$

where $\bar{V}_f$, $\bar{r}_f$, $\bar{i}_f$ and $\bar{\varphi}_f$ are variables in physical units. Now from equation (7) using the variables in p.u. the field flux dynamics can be represented as

$$\frac{1}{\omega_b}\frac{d\varphi_f}{dt} = -r_f i_f + V_f. \tag{8}$$

Substituting in equation (8) the flux expression for i_f obtained from equation (4), gives

$$T'_{do}\dot{\varphi}_f = \bar{a}_{31}\varphi_f + \bar{a}_{32}\varphi_{kd} + \bar{a}_{33}V_d + \bar{b}_3 V_f, \tag{9}$$

where $T'_{do} = (1/\omega_b)/(L_f/r_f)$ and T'_{do} is expressed in seconds. In the same way, equations for dynamics of ϕ_g, ϕ_{kd} and ϕ_{kq} can be obtained, respectively

$$T'_{qo}\dot{\varphi}_g = \bar{b}_{11}\varphi_g + \bar{b}_{12}\varphi_{kq} + \bar{b}_{13}V_q \tag{10}$$
$$T''_{do}\dot{\varphi}_{kd} = \bar{b}_{21}\varphi_f + \bar{b}_{22}\varphi_{kd} + \bar{b}_{23}V_d \tag{11}$$
$$T''_{qo}\dot{\varphi}_{kq} = \bar{b}_{31}\varphi_g + \bar{b}_{32}\varphi_{kq} + \bar{b}_{33}V_q, \tag{12}$$

where the time constants $T'_{qo} = (1/\omega_b)/(L_g/r_g)$, $T''_{do} = (1/\omega_b)/(L'_{kd}/r_{kd})$ and $T'_{qo} = (1/\omega_b)/(L'_{kq}/r_{kq})$ are expressed in seconds.

2.2.2. Per unit stator basic equations. Following the same procedure as in subsection 2.2.1, using the equations (3)–(4) and equations (9)–(12), the scale per unit dynamic equations for the stator flux ϕ_d and ϕ_q can be rewritten as

$$\frac{1}{\omega_b}\dot{\varphi}_d = \frac{\omega}{\omega_b}\varphi_q + r_s i_d + V_d \tag{13}$$
$$\frac{1}{\omega_b}\dot{\varphi}_q = -\frac{\omega}{\omega_b}\varphi_d + r_s i_q + V_q. \tag{14}$$

The coefficients in equations (9)–(14) are constants calculated using the plant parameters, and are presented in per unit except the time which is in seconds and the angular velocity which is in radians/seconds, in order to achieve the singularly perturbed form.

2.3. *Complete model*

After some routine calculations from equation (1) to equation (14), we obtain the following model of synchronous generator of the 8th order

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \end{bmatrix} = \begin{bmatrix} F_1(\mathbf{x}_1, \mathbf{x}_2, \mathbf{z}, T_m) \\ F_2(\mathbf{x}_1, \mathbf{x}_2, \mathbf{z}) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} V_f \tag{15}$$

$$\mu\dot{\mathbf{z}} = F_3(\mathbf{x}_1, \mathbf{x}_2, \mathbf{z}), \tag{16}$$

where $\mathbf{x}_1=(x_1,x_2,x_3)^T$, $\mathbf{x}_2=(x_4,x_5,x_6)^T$, $\mathbf{z}=(z_1,z_2)^T$, $x_1=\delta$, $x_2=\omega$, $x_3=\phi_f$, $x_4=\phi_g$, $x_5=\phi_{kd}$, $x_6=\phi_{kq}$, $z_1=\phi_d$, $z_2=\phi_q$, $\mu=(1/\omega_b)$,

$$F_1=\begin{bmatrix} x_2-\omega_b \\ d_mT_m-(a_{21}x_3z_2+a_{22}x_4z_1+a_{23}x_5z_2+a_{24}x_6z_1+a_{25}z_1z_2) \\ a_{31}x_3+a_{32}x_5+a_{33}z_1 \end{bmatrix},$$

$$F_2=\begin{bmatrix} b_{11}x_4+b_{12}x_6+b_{13}z_2 \\ b_{21}x_3+b_{22}x_5+b_{23}z_1 \\ b_{31}x_4+b_{32}x_6+b_{33}z_2 \end{bmatrix},$$

$$F_3=\begin{bmatrix} c_{11}x_2x_4+c_{12}x_2x_6+c_{13}x_2z_2+c_{14}z_1+c_{15}\sin x_1 \\ c_{21}x_2x_3+c_{22}x_2x_5+c_{23}x_2z_1+c_{24}z_2+c_{25}\cos x_1 \end{bmatrix},\quad B_1=\begin{bmatrix} 0\\0\\b_3 \end{bmatrix} \text{ and } B_2=0.$$

The coefficients of equations (15)–(16) depend on the plant parameters.

3. Singularly perturbed approach

3.1. *Singularly perturbed model*

In this paper we are dealing with the singularly perturbed model having the form

$$\frac{dx}{dt}=f(x,z,\mu,u),\quad x(0)=x_0 \tag{17}$$

$$\mu\frac{dz}{dt}=g(x,z,\mu),\quad z(0)=z_0, \tag{18}$$

where $x\in R^n$, $z\in R^m$, $u\in R$, $\mu\in R$; f and g are smooth functions of their arguments and linear on z and u, $\mu>0$ is a small parameter, and μ is a bounded control vector

$$|u|\le u_0\quad\text{with } u_0>0. \tag{19}$$

3.2. *Control design procedure*

The sliding mode control design procedure for original system equations (17)–(18) consists of two steps.

Step 1: Setting $\mu=0$ makes instantaneous the fast dynamics equation (18)

$$0=g(x,z,0). \tag{20}$$

Suppose that there exists a smooth isolated solution of equation (20) with respect z in the form

$$\bar{z}=h(x) \tag{21}$$

where $\bar{z}$ presents the quasi-steady state. Substituting equation (21) in equation (17), we obtain the reduced order model (ROM)

$$\frac{d\bar{x}}{dt}=f(\bar{x},h(\bar{x}),0,u) \tag{22}$$

where $\bar{x}(t)$ defines the solution of equation (22) for a fixed control $u(\bar{x})$.

Remark 1: Taking into account the specific feature of the synchronous generator model equations (15)–(16), we assume that equation (20) is linear with respect to and u, and a corresponding solution equation (21) do exist. Consequently, the ROM equation (22) is linear on u.

Step 2: Design a non-linear sliding surface $s(\bar{x})=0$, $s\in R$ for the system (22), such that there is a solution of the equation

$$\frac{ds}{dt}=\bar{G}f\left(\bar{x},h(\bar{x}),0,u_{\text{eq}}\right)=0,\quad \bar{G}=\left\{\frac{ds}{d\bar{x}}\right\}$$

with respect to the equivalent control, $u_{\text{eq}}(\bar{x})$ (Utkin 1992), and the sliding mode equation (SME)

$$\frac{d\bar{x}}{dt}=f\left(\bar{x},h(\bar{x}),0,u_{\text{eq}}(\bar{x})\right) \tag{23}$$

$$s(\bar{x})=0 \tag{24}$$

has the desired properties. Select a discontinuous control

$$u(\bar{x})=\begin{cases} u^+(\bar{x}) & \text{if}\quad s(\bar{x})>0,\quad \left|u^+(\bar{x})\right|\le u_0 \\ u^-(\bar{x}) & \text{if}\quad s(\bar{x})<0,\quad \left|u^-(\bar{x})\right|\le u_0 \end{cases}, \tag{25}$$

that makes the sliding surface equation (24) to be attractive.

Note that the synchronous machine model equations (15)–(16) is a particular case of equations (17)–(18), when the functions f and g depending on z and u linearly and consequently the solution of system equations (17)–(18) in Filippov sense exists at least for a small t (see for example Utkin 1992). Moreover, from equation (24) one of the vector $\bar{x}$ components can be expressed as a function of other $(n-1)$ components. Therefore, in fact, SME equation (23) has the order $(n-1)$. So, the order of the original system equations (17)–(18), is reduced first, by using the motion separation due to different time scale, and second, via sliding mode.

To justify the proposed control design (TSCD) procedure (see steps 1 and 2), first we will analyse the behaviour of the original SPRCS equations (17)–(18) and equation (25) when the state vector crosses the switching surface, and then investigate the entrance of SPRCS solutions into the sliding mode domain (see subsection 3.3). Finally, the stability condition for original SPRCS will be derived (see subsection 3.4).

3.3. *Analysis of SPRCS solutions crossing sliding surface*

In this subsection we will study the behaviour of the original closed-loop system (SPRCS) equations (17)–(18) and equation (25) out from sliding mode domain. If a solution of the SPRCS is not crossing the discontinuity surface equation (24), it can be analysed by the classical method of singular perturbations (see Vasil'eva *et al.* 1995, Kokotovic *et al.* 1986). On the other hand, the specific feature of SPRCS describing the behaviour of synchronous machines is that the equations of slow variables depend on the relay control equation (25), and consequently after a finite number of switches the trajectory of original SPRCS will enter into the sliding mode domain. In this subsection we will show that in the case of finite switches we can use the reduced order model to describe the slow motions in the SPRCS. Doing so, we have to describe specific features of SPRCS for both domains $s > 0$ and $s < 0$. Moreover, it is necessary to verify the attraction condition for the switching point.

Denote the domains of definition for variables z and x as Z and X, respectively. The discontinuity surface $s(x) = 0$ divides the domains X into the parts defined as X^- for $s < 0$ and X^+ for $s > 0$, respectively; and define the system structure as

$$f^+(x, z, \mu) = f\big(x, z, \mu, u^+(x)\big) \quad \text{for } s \geq 0 \quad \text{and} \quad u = u^+(x)$$

and

$$f^-(x, z, \mu) = f(x, z, \mu, u^-(x)) \quad \text{for } s \leq 0 \quad \text{and} \quad u = u^-(x)$$

with

$$f^+ \in C^2\big[\bar{X}^+ \times [0, \mu_0]\big], \quad f^- \in C^2\big[\bar{X}^- \times [0, \mu_0]\big].$$

3.3.1. System in the domain $s < 0$.

Denote

$$\frac{ds^-}{dt}(x, z, \mu) = Gf^-(x, z, \mu), \quad \frac{ds^+}{dt}(x, z, \mu) = Gf^+(x, z, \mu).$$

Suppose that $x_0 \in X^-$, $z_0 \in Z$. It is natural to assume that for the original system equations (17)–(18) and (25), the following conditions of the Tikhonov theorem hold (see, for example, Vasil'eva *et al.* 1995).

[a1] The function $\bar{z} = h(\bar{x})$ is an isolated solution of $0 = g(x, z, 0)$ for all $x \in X$.

[a2] The Cauchy problem for slow dynamics

$$\frac{d\bar{x}^-}{dt} = f^-(\bar{x}^-, h(\bar{x}^-), 0), \quad \bar{x}^-(0) = x_0 \tag{26}$$

has a unique solution $\bar{x}^-(t)$ on $[0, \bar{t}_s]$, where $\bar{t}_s$ is the switching point i.e. the smallest root of the equation $s\big(\bar{x}^-(\bar{t}_s)\big) = 0$.

[a3] The equilibrium point $\Pi z = 0$ of the system

$$\frac{d(\Pi z)}{d\tau} = g(\bar{x}^-, \Pi z + h(\bar{x}^-), 0)$$

where $\Pi z = z - h(\bar{x})$, $\tau = (t/\mu)$ is asymptotically stable, moreover,

$$\operatorname{Re}\operatorname{Spec}\frac{\partial g}{\partial \Pi z}(\bar{x}^-, h(\bar{x}^-), 0) < -\alpha, \quad \alpha > 0, \quad \text{for all } x \in X.$$

[a4] The trajectory of the reduced system equation (26) crosses the switching surface $s(x) = 0$, without tangential touch, i.e.

$$\frac{ds^-}{dt} = \bar{G}f^-\big(\bar{x}^-(\bar{t}_s), h\big(\bar{x}^-(\bar{t}_s)\big), 0\big) > 0.$$

Remark 2: In the system equations (15)–(16) the function g depends on the fast variable z linearly. This implies that every initial value of equation (18) belongs to the domain of attraction for the equilibrium point $\Pi z = 0$.

Now from Vasil'eva theorem (Vasil'eva *et al.* 1995) and implicit function theorem it follows that for sufficiently small μ there exists a time moment $t = t_s(\mu)$ such that for the slow coordinate of the original SPRCS we have

$$s(x(t_s(\mu), \mu)) = 0$$

and moreover, for all $\mu \in [0, \mu_0]$

$$\frac{ds^-}{dt} = Gf^-(x(t_s(\mu), \mu), z(t_s(\mu), \mu), \mu) > 0, \quad G = \left\{\frac{ds}{dx}\right\}.$$

Now we have to consider two alternative variants for SPRCS solution behaviour

- a solution of original SPRCS will enter into the domain $X^+ \times Z$;
- a solution of original SPRCS will enter into the sliding domain.

3.3.2. Entrance into the domain $s > 0$. From the condition [a4] and the boundary layer method (Vasil'eva *et al.* 1995), it follows that a solution of the original SPRCS will reach the switching surface $s(x(t_s(\mu),\mu)) = 0$ at the switching point

$$\begin{aligned}&(x(t_s(\mu), \mu), z(t_s(\mu), \mu))\\ &\quad = \left(\bar{x}^-(\bar{t}_s) + \mathrm{O}(\mu), h\left(\bar{x}^-(\bar{t}_s)\right) + \mathrm{O}(\mu)\right).\end{aligned}$$

This means that the point $(x(t_s(\mu),\mu),\ z(t_s(\mu),\mu))$ does not belong to the sliding mode domain, and the solution of equations (17)–(18) and equation (25) will enter into the domain $X^+ \times Z$. We can consider the coordinate of the switching point $(x(t_s(\mu),\mu),\ z(t_s(\mu),\mu))$ as the initial condition for SPRCS into the domain $X^+ \times Z$ and suppose that for the original SPRCS, in the domain $X^+ \times Z$ the following conditions are satisfied:

[b1] $\dfrac{ds^+}{dt} = \bar{G}f^+\left(\bar{x}^-(\bar{t}_s), h\left(\bar{x}^-(\bar{t}_s)\right), 0\right) > 0$

then from the Tikhonov theorem it follows that for sufficiently small μ

$$\frac{ds^+}{dt} = Gf^+(x(t_s(\mu), \mu), z(t_s(\mu), \mu), \mu) > 0.$$

[b2] Suppose that the Cauchy problem

$$\frac{d\bar{x}^+}{dt} = f^+\left(\bar{x}^+, h\left(\bar{x}^+\right), 0\right), \quad \bar{x}^+\left(\bar{t}_s\right) = \bar{x}^-\left(\bar{t}_s\right)$$

has a unique solution on $\left[\bar{t}_s, T\right]$.

The following Lemma is true (Fridman 2002a):

Lemma 1: *Suppose that the original SPRCS equations* (17)–(18) *and equation* (25) *satisfies the conditions* [a1]–[a4] *and* [b1]–[b2]. *Then there exists small* $\mu_0 > 0$ *such that for all* $\mu \in [0, \mu_0]$ *there is a unique solution* $(x(t,\mu),\ z(t,\mu))$ *of Cauchy problem equations* (17)–(18) *on* $[0, T]$, *and*

$$\lim_{\mu\to 0} x(t, \mu) = \bar{x}(t) = \begin{cases} \bar{x}^-(t) & \text{for} \quad t \in \left[0, \bar{t}_s\right] \\ \bar{x}^+(t) & \text{for} \quad t \in \left[\bar{t}_s, T\right] \end{cases},$$

$$\lim_{\mu\to 0} z(t, \mu) = h(\bar{x}(t)) \quad \text{for } t \in (0, T].$$

Remark 3: In the same way, we can prove that it is possible to use the equations for slow motions to analyse the system equations (17)–(18) and equation (25) dynamics in the case when the solution leaves the domain $X^+ \times Z$ and enters into the domain $X^- \times Z$.

3.3.3. Transition into sliding domain. In this subsection the behaviour of the original SPRCS equations (17)–(18) and equation (25) into the sliding domain, is described. Denote as

$$S_0 = \left\{x : \frac{ds^-}{dt}(\bar{x}, h(\bar{x}), 0) > 0, \frac{ds^+}{dt}(\bar{x}, h(\bar{x}), 0) < 0\right\},$$

$$S_\mu = \left\{(x, z, \mu) : \frac{ds^-}{dt}(x, z, \mu) > 0, \frac{ds^+}{dt}(x, z, \mu) < 0\right\}$$

the sliding domains for the systems equation (22) and equations (17)–(18), respectively. Suppose that the control resources allow us to achieve the following sliding mode existence conditions (Utkin 1992):

[c1] $\dfrac{ds^-}{dt}\left(\bar{x}^-(\bar{t}_s), h\left(\bar{x}^-(\bar{t}_s)\right), 0\right) > 0,$
$\dfrac{ds^+}{dt}\left(\bar{x}^-(\bar{t}_s), h\left(\bar{x}^-(\bar{t}_s)\right), 0\right) < 0.$

Now from the Tikhonov theorem it follows that for sufficiently small μ

$$\frac{ds^-}{dt} = Gf^-(x(t_s(\mu)), z(t_s(\mu)), \mu) > 0 \quad \text{and}$$
$$\frac{ds^+}{dt} = Gf^+(x(t_s(\mu)), z(t_s(\mu)), \mu) < 0.$$

This means that, a solution of the original system equations (17)–(18) and equation (25) enter into the sliding domain S_μ without tangential touch. Therefore, we can consider the coordinate of the switching point $(x(t_s(\mu),\mu),\ z(t_s(\mu),\mu))$ as the initial condition for SPRCS into the sliding domain S_μ. Hence, a solution of the Cauchy problem equations (17)–(18) with equation (25) into S_μ is described by the following system (Utkin 1992):

$$\left.\begin{aligned} &\frac{dx^*}{dt} = f\big(x^*, z^*, \mu, u_{eq}(x^*, z^*, \mu)\big), \quad \mu\frac{dz^*}{dt} = g(x^*, z^*, \mu) \\ &x^*(t_s(\mu), \mu) = x(t_s(\mu), \mu), \\ &\qquad z^*(t_s(\mu), \mu) = z(t_s(\mu), \mu), \quad s(x^*) = 0, \end{aligned}\right\} \tag{27}$$

where $t \in [t_0(\mu), T]$, $x^* \in R^{n-1}$, $z^* \in R^m$, $u \in R$, $\mu \in [0, \mu_0]$, and $u_{\text{eq}}(x^*, z^*, \mu)$ is the equivalent control calculated as a solution of

$$\frac{ds}{dt} = Gf\big(x^*, z^*, \mu, u_{eq}\big) = 0, \quad s(x^*) = 0. \tag{28}$$

Similar to the above case (§ 3.3.2) we suppose that for the system (27)–(28) the following conditions of the Tikhonov theorem hold:

[c2] The reduced (by $\mu = 0$) sliding mode equation

$$\frac{d\bar{x}^*}{dt} = f\big(\bar{x}^*, h(\bar{x}^*), 0, \bar{u}_{eq}(\bar{x}^*)\big), \quad \bar{x}^*(\bar{t}_s) = x_0^*$$

with $\bar{u}_{eq}(\bar{x}^*) = u_{eq}(\bar{x}^*, h(\bar{x}^*), 0)$ has a unique solution $\bar{x}^*(t)$ on $[\bar{t}_s, T]$, and $\bar{x}^*(t) \in S_0$ for all $t \in [\bar{t}_s, T]$.

The following Lemma is true (Fridman 2002c):

Lemma 2: *Suppose that the original SPRCS equations* (17)–(18) *and* (25) *satisfies the conditions* [a1]–[a4] *and* [c1]–[c2]. *Then there exists a small* $\mu_0 > 0$ *such that for all* $\mu \in [0, \mu_0]$ *there is a unique solution* $(x(t,\mu), z(t,\mu))$ *of equations* (17)–(18) *and equation* (25) *on* $[0, T]$ *and*

$$\lim_{\mu\to 0} u_{eq}(x(t, \mu), z(t, \mu), \mu) = \bar{u}_{eq}(\bar{x}^*(t)), \quad \text{for } t \in \big[\bar{t}_s, T\big], \tag{29}$$

$$\lim_{\mu\to 0} x(t, \mu) = \bar{x}(t) = \begin{cases} \bar{x}^-(t) & \text{for } t \in \big[0, \bar{t}_s\big] \\ \bar{x}^*(t) & \text{for } t \in \big[\bar{t}_s, T\big] \end{cases}, \tag{30}$$

$$\lim_{\mu\to 0} z(t, \mu) = h(\bar{x}(t)) \quad \text{for} \quad t \in (0, T]. \tag{31}$$

Remark 4: If a solution of equations (17)–(18) and equation (25) will leave the sliding modes then it will not affect the zero approximation of the fast and the slow dynamics equations, since in this case the slow motion integral manifold is continuous (Fridman 2002c).

3.4. *Stability analysis*

Consider the case when the original SPRCS has an equilibrium into S_μ. Solving equation (28) for $u^*_{\text{eq}}(x(t, \mu), z(t, \mu))$ and substituting it in equation (17), we obtain the *smooth* algebraic-differential system described the sliding mode dynamics. Express one coordinate of x as a function of other $(n-1)$ coordinates. Then a sliding mode dynamics are governed by the following singularly perturbed $(n+m-1)$th order system:

$$\frac{dx^\otimes}{dt} = f^\otimes\big(x^\otimes, z^\otimes, \mu\big) \quad \mu\frac{dz^\otimes}{dt} = g^\otimes\big(x^\otimes, z^\otimes, \mu\big), \tag{32}$$

where the vector $x^\otimes \in R^{n-1}$ consists of the $(n-1)$-independent coordinates of x, $z^\otimes = z$, $g^\otimes$, and $f^\otimes \in R^{n-1}$ are the values of g and the corresponding component of f computed at $u = u_{\text{eq}}(x^\otimes, z^\otimes, \mu)$. For the case of synchronous machine the equation $g^\otimes(\bar{x}^\otimes, h^\otimes(\bar{x}^\otimes), 0) = 0$ has a unique solution $\bar{z}^\otimes = h^\otimes(\bar{x}^\otimes)$, consequently the slow dynamics in equation (32) are described by the system

$$\frac{dx^\otimes}{dt} = \bar{f}^\otimes\big(\bar{x}^\otimes\big) = f^\otimes\big(\bar{x}^\otimes, h^\otimes\big(\bar{x}^\otimes\big), 0\big) \quad 0 = g^\otimes\big(\bar{x}^\otimes, h^\otimes\big(\bar{x}^\otimes\big), 0\big). \tag{33}$$

Let us denote $x^\otimes_{eq}$ as an equilibrium point of equation (33). Then from the Klimushchev–Krasovskii theorem (Klimushchev and Krasovskii 1962), Theorem 1 follows.

Theorem 1: *Suppose that the equilibrium point of the system equations* (17)–(18) *belongs to* S_μ, $x^\otimes_{eq} \in S_0$, *and*

$$\frac{\partial \bar{f}^\otimes}{\partial x^\otimes}\Big(x^\otimes_{eq}, h^\otimes\Big(x^\otimes_{eq}\Big), 0\Big) \tag{34}$$

$$\frac{\partial g^\otimes}{\partial z^\otimes}\Big(x^\otimes_{eq}, h^\otimes\Big(x^\otimes_{eq}\Big), 0\Big) \tag{35}$$

are Hurwitz matrices. Then there exists $\mu_0 > 0$ *such that the equilibrium point to the system equations* (17)–(18) *is uniformly asymptotically stable with respect to* $\mu \in [0, \mu_0]$.

Therefore we can conclude that in order to verify correctness of the proposed control design procedure (steps 1 and 2) it is enough to check the conditions presented in the subsection 3.2–3.3.

4. Control of generator

In this section, following the proposed in section 3 control design procedure, we will derive a reduced model which describes the generator slow dynamics. Then, based on this a discontinuous control law for the generator will be designed, and the conditions presented in section 3 will be checked.

4.1. *Reduced model of synchronous machine*

The fast dynamics equation (16) rewritten as

$$\mu \dot{z}_1 = c_{11}x_2x_4 + c_{12}x_2x_6 + c_{13}x_2z_2 + c_{14}z_1 + c_{15}\sin x_1 \quad (36)$$

$$\mu \dot{z}_2 = c_{21}x_2x_3 + c_{22}x_2x_5 + c_{23}x_2z_1 + c_{24}z_2 + c_{25}\cos x_1 \quad (37)$$

is linear with respect to fast variables and it can be neglected by making $\mu = 0$, that is

$$0 = A_R z + F_R, \quad (38)$$

where

$$A_R = \begin{bmatrix} c_{14} & c_{13}x_2 \\ c_{23}x_2 & c_{24} \end{bmatrix},$$

$$F_R = \begin{bmatrix} c_{11}x_2x_4 + c_{12}x_2x_6 + c_{15}\sin x_1 \\ c_{21}x_2x_3 + c_{22}x_2x_5 + c_{25}\cos x_1 \end{bmatrix}, \text{ and } rank\ A_R = 2.$$

The fast dynamics equations (36)–(37) do not depend on the control u directly, and the matrix A_R has full rank, that is why a solution of equation (38) for z_1 and z_2 can be calculated as

$$\mathbf{z} = -A_R^{-1}F_R := h(\mathbf{x}_1, \mathbf{x}_2) := \begin{bmatrix} h_1(\mathbf{x}_1, \mathbf{x}_2) \\ h_2(\mathbf{x}_1, \mathbf{x}_2) \end{bmatrix}. \quad (39)$$

So, the condition [a1] is satisfied. Substitution of equation (39) into equation (15) gives the following reduced (6th order) model:

$$\begin{bmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \end{bmatrix} = \begin{bmatrix} \bar{F}_1(\mathbf{x}_1, \mathbf{x}_2, T_m, h_1(\mathbf{x}_1, \mathbf{x}_2), h_2(\mathbf{x}_1, \mathbf{x}_2)) \\ \bar{F}_2(\mathbf{x}_1, \mathbf{x}_2, h_1(\mathbf{x}_1, \mathbf{x}_2), h_2(\mathbf{x}_1, \mathbf{x}_2)) \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} V_f, \quad (40)$$

where

$$\bar{F}_1 = \begin{bmatrix} x_2 - \omega_b \\ d_mT_m - [(a_{22}x_4 + a_{24}x_6)\cdot h_1(\cdot) + (a_{21}x_3 + a_{23}x_5)\cdot h_2(\cdot) + a_{25}h_1(\cdot)h_2(\cdot)] \\ a_{41}x_3 + a_{42}x_4 + a_{43}x_5 + a_{44}x_6 + a_{45}\sin x_1 + a_{46}\cos x_1 \end{bmatrix},$$

$$\bar{F}_2 = \begin{bmatrix} b_{11}x_4 + b_{12}x_6 + b_{13}h_2(\mathbf{x}_1, \mathbf{x}_2) \\ b_{21}x_3 + b_{22}x_5 + b_{23}h_1(\mathbf{x}_1, \mathbf{x}_2) \\ b_{31}x_4 + b_{32}x_6 + b_{33}h_2(\mathbf{x}_1, \mathbf{x}_2) \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 0 \\ b_3 \end{bmatrix}.$$

4.2. *Angular speed control*

The first subsystem of equation (40) has the Nonlinear Block Controllable form consisting of three blocks. Therefore in order to design the non-linear sliding surface we use the block control technique (Loukianov 1998). To satisfy the control objective, namely, rotor angle stability enhancement, we define the control error as

$$\varsigma_2 = x_2 - \omega_b. \quad (41)$$

The time derivative of equation (41) along the trajectories of equation (40), takes the form

$$\dot{\varsigma}_2 = f_2(\mathbf{x}_1, \mathbf{x}_2, T_m) + b_2(\mathbf{x}_1, \mathbf{x}_2)x_3 \quad (42)$$

where $f_2 = d_mT_m - (a_{22}x_4h_1(\cdot) + a_{23}x_5h_2(\cdot) + a_{24}x_6h_1 \times (\cdot) + a_{25}h_1(\cdot)h_2(\cdot))$, $b_2 = a_{21}h_2(\cdot)$, and $b_2(t)$ is a positive function of the time. To introduce a new desired behaviour we put

$$x_3 = -[b_2(\mathbf{x}_1, \mathbf{x}_2)]^{-1}[f_2(\mathbf{x}_1, \mathbf{x}_2, T_m) + k_0\varsigma_2 - s_\omega], \quad k_0 > 0. \quad (43)$$

Then using equation (43) the switching surface can be defined as

$$s_\omega = 0, \quad s_\omega = b_2(\mathbf{x}_1, \mathbf{x}_2)x_3 + f_2(\mathbf{x}_1, \mathbf{x}_2, T_m) + k_0(x_2 - \omega_b). \quad (44)$$

The projection motion on the subspace s_ω can be derived using equation (44) and equation (40) of the form

$$\dot{s}_\omega = f_s(\mathbf{x}_1, \mathbf{x}_2, T_m) + b_s(\mathbf{x}_1, \mathbf{x}_2)V_f,$$

where f_s is a continuous function, $b_s(\cdot) = b_3b_2(\cdot)$, and $b_s(t)$ is a positive function of the time.

4.3. *Singularly perturbed analysis of synchronous generator*

4.3.1. Stability analysis outside the sliding surface. Let us show the conditions of Lemmas 1 and 2 and Tikhonov Theorem in the synchronous generator control problem. The matrix A_R in equation (38) for all real synchronous generators known for the authors is Hurwitz. This ensures that the conditions [a1]–[a3] and (35) are satisfied.

4.3.2. Sliding mode existence. Design the relay control law as follows:

$$V_f = -u_0 \operatorname{sign}(s_\omega), \quad u_0 > 0. \tag{45}$$

Then we can see that condition

$$u_0 \geq \left| V_{feq}(\mathbf{x}_1, \mathbf{x}_2, T_m) \right|, \quad V_{feq} = [b_s(\mathbf{x}_1, \mathbf{x}_2)]^{-1} f_s(\mathbf{x}_1, \mathbf{x}_2, T_m),$$

ensures the existence of the sliding domains with nonzero measure for the systems equation (16) and equation (40) with control equation (45). This means that the assumptions [a4], [b1] or [c1] are true for all solutions to the system equation (16) excluding the set of initial condition of zero measure, and after some time t_s a sliding mode motion occurs.

4.3.3. Slow dynamics stability. Once the sliding mode motion is achieved, this motion is governed by the reduced order system

$$\dot{\varsigma}_2 = -k_0 \varsigma_2 \tag{46}$$

$$\dot{\mathbf{x}}_2 = \bar{F}_2(\mathbf{x}_1, \mathbf{x}_2, h_1(\mathbf{x}_1, \mathbf{x}_2), h_2(\mathbf{x}_1, \mathbf{x}_2)) \tag{47}$$

where the linear subsystem equation (46) describing the linearized mechanical dynamics, has the desired eigenvalue $-k_0$, while equation (47) represents the rotor flux dynamics. The subsystem equation (46) is asymptotically stable, that is, $\lim_{t\to\infty} \varsigma_2(t) = 0$ and the angle $x_1(t) = x_1(0) + \int_0^t \varsigma_2(\upsilon)d\upsilon$ tends to a steady state $x_{1ss} = \delta_{ss}$ as the control error $\varsigma_2(t)$ tends to zero. On the invariant subspace $\{\xi = (x_{1ss}, 0, 0)^T, \quad \mathbf{x}_2 \in R^3\}$, where $\xi = (x_1, \varsigma_2, s_\omega)^T$, the dynamics of $\mathbf{x}_2$ are referred to as the *zero dynamics*. To derive this dynamics, we put $x_1 = \delta_{ss}$ and $x_2 = \omega_b$, then equation (47) can be rewritten as a linear system with non-vanishing perturbation

$$\dot{\mathbf{x}}_2 = A_{sm}\mathbf{x}_2 + F_{sm}(\mathbf{x}_2, \delta_{ss}, \omega_b, T_m)$$

where

$$A_{sm} = \begin{bmatrix} b_{11} - b_{13}ar_{21}c_{11} & -b_{13}ar_{22}c_{22} & b_{12} - b_{13}ar_{21}c_{12} \\ -b_{23}ar_{11}c_{11} & b_{22} - b_{23}ar_{12}c_{22} & -b_{23}ar_{11}c_{12} \\ b_{31} - b_{33}ar_{21}c_{11} & -b_{33}ar_{22}c_{22} & b_{32} - b_{33}ar_{21}c_{12} \end{bmatrix},$$

and $F_{sm}(\mathbf{x}_2, \delta_{ss}, \omega_b, T_m)$ is a bounded non-linear function. The matrix A_{sm} is Hurwitz by virtue of rotor flux dynamics (see plant parameters). Hence, a solution of equations (46)–(47) can exponentially converges to a steady state $\mathbf{x}_2 = \mathbf{x}_{2ss}$ defined by value of T_m (Loukianov *et al*. 2000, Khalil 1996). So condition (34) is satisfied.

5. Simulation results

The performance of the proposed control algorithm was tested on the complete 8th order model of synchronous generator connected to an infinite bus through a transmission line, figure 1.

The parameters of the synchronous machine and external network in p.u. are

$$L_d = 1.81, \quad L'_d = 0.3, \quad L''_d = 0.23, \quad L_q = 1.76,$$
$$L'_q = 0.6, \quad L_{\text{ext}} = 0.1, \quad R_{\text{ext}} = 0.001,$$
$$T'_{do} = 8.0 \text{ sec}, \quad T'_{qo} = 1.0 \text{ sec}, \quad T''_{do} = 0.03 \text{ sec},$$
$$T''_{qo} = 0.07 \text{ sec}.$$

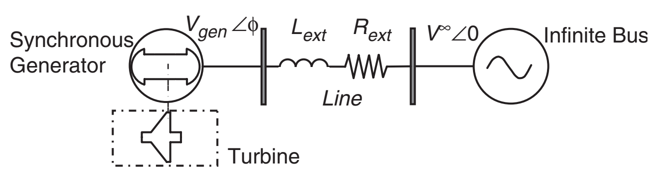

Figure 1. Single machine connected to an external network infinite bus.

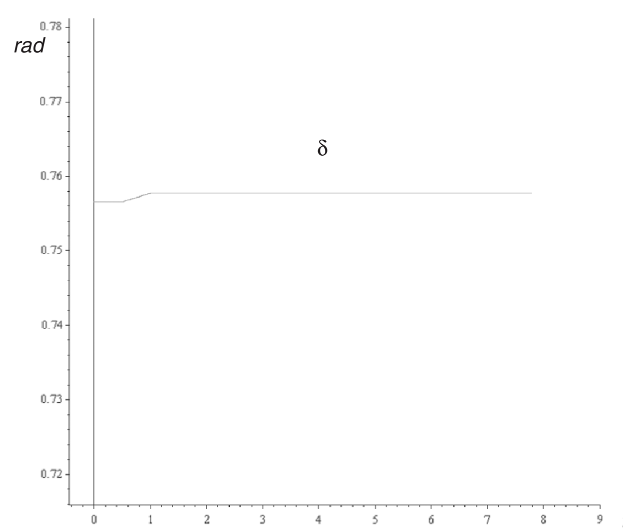

Figure 2. Power angle affected by a mechanical perturbation.

For these parameters we obtain the parameters of mathematical model equations (15)–(16). The controller gain was adjusted to $k_0 = 10$. The eigenvalues of A_{sm} was calculated as, $\lambda_4 = -38.77$, $\lambda_5 = -0.5024$ and $\lambda_6 = -27.04$. Figures 3–10 depict results under two different events

(a) in $t = 0.5\,s$, T_m experienced a pulse increment of 7% for $0.5\,s$, (figures 3 to 5 and 9), and
(b) in $t = 1.7\,s$, a three-phase short circuit for a period of $150\,ms$ is simulated at the transformer terminals (figures 6–10).

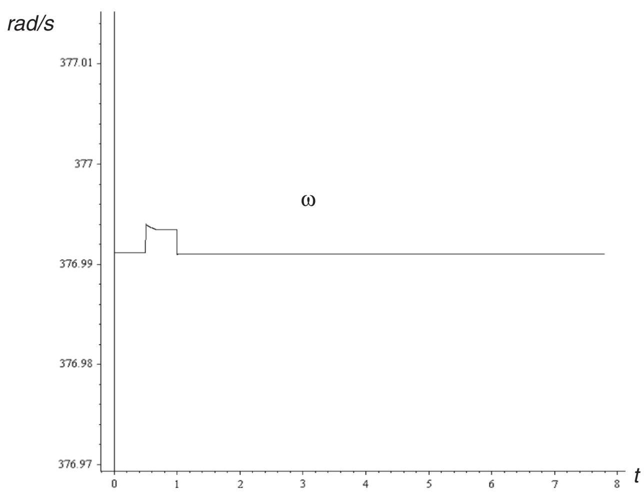

Figure 3. Rotor velocity affected by a mechanical perturbation.

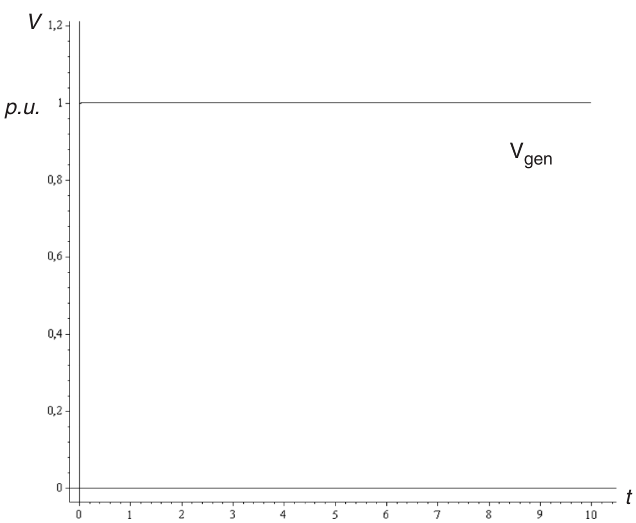

Figure 4. Generator voltage affected by a mechanical perturbation.

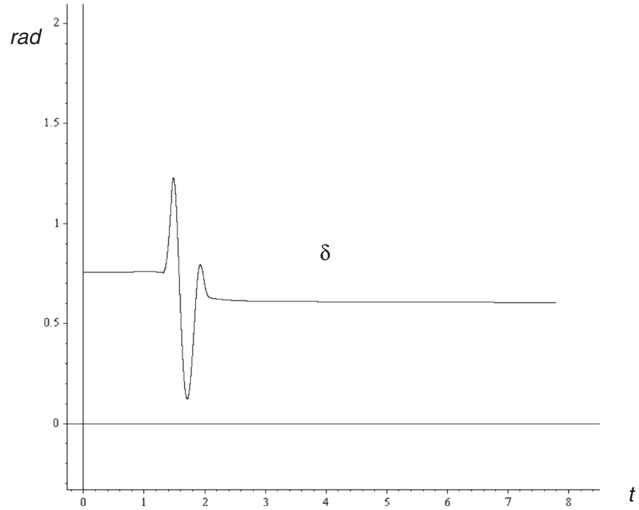

Figure 5. Power angle affected by a 0.15 s short circuit.

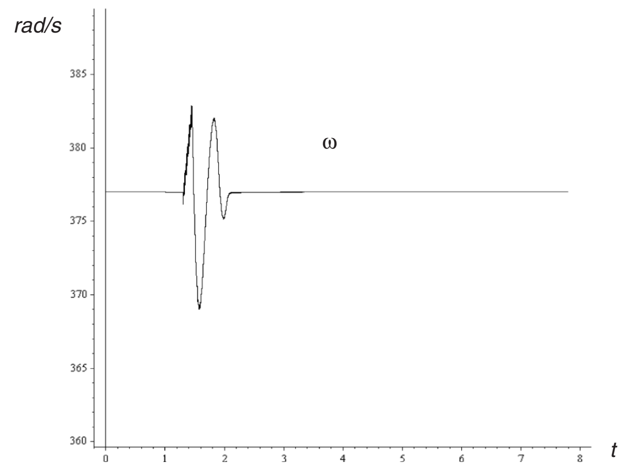

Figure 6. Rotor angular velocity affected by a 0.15 s short circuit.

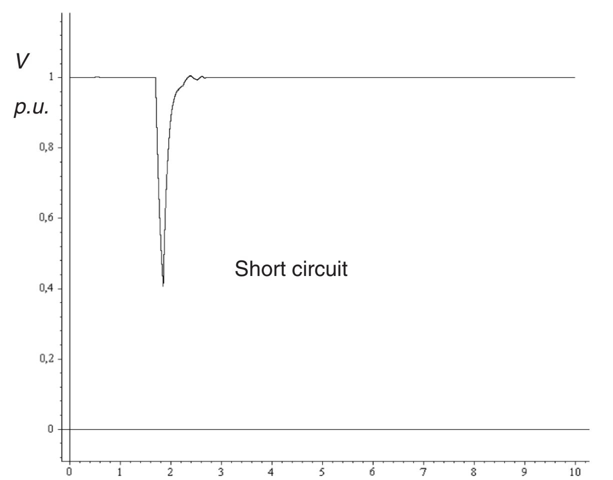

Figure 7. Generator voltage affected by a 0.15 s short circuit.

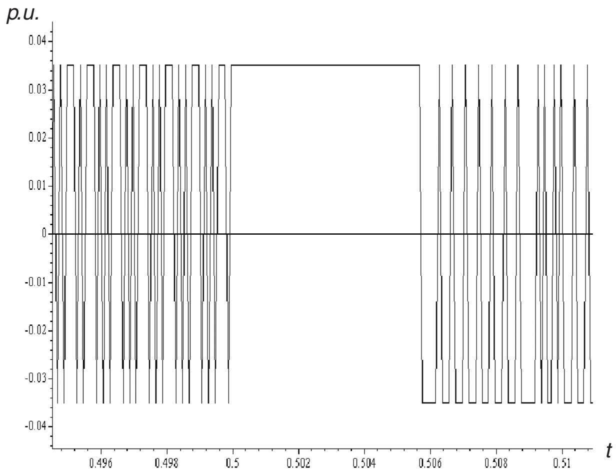

Figure 8. Voltage Vf performance.

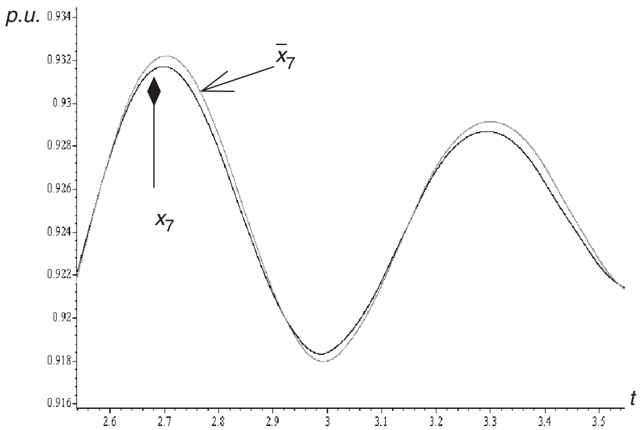

Figure 9. Natural response of high motions.

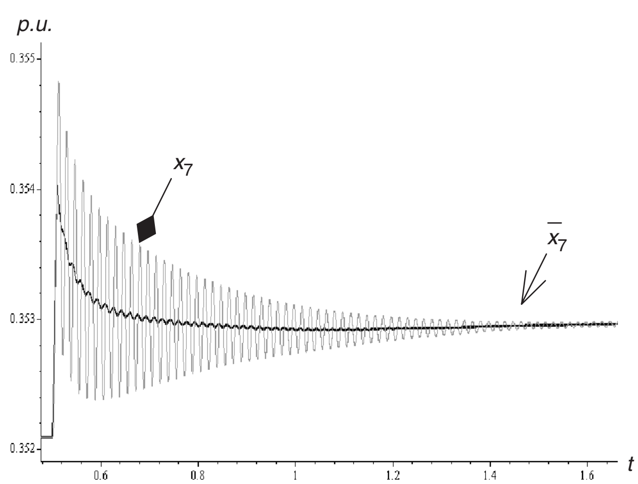

Figure 10. Short circuit fast motion response: Full model.

These figures reveal some important aspects:

1. State variables hastily reach a steady state condition after small and large disturbances, exhibiting the stability of the closed-loop system.
2. The terminal voltage recovers their steady state value after the short circuit.

6. Conclusions

In this paper the possibility of apply a sliding mode control algorithm for non-linear SPRCS described the synchronous generator model dynamics is analysed. The main specific feature of synchronous machine models is the following: the actuator is inside the field flux dynamics. For SPRCS described the behaviour of synchronous machines this means that the slow equations depend on relay control. For such system the following two steps control design (TSCD) are proposed: firstly, the natural two scale properties of synchronous generator are used to obtain the reduced order model, and then based on this reduced model a sliding mode control algorithm ensuring the desired behaviour of the generator is designed. The effectiveness of proposed algorithm is illustrated by simulations with the parameters of a real generator.

Proposed control ideology could be useful for complete models of electrical machines networks because the corresponding SPRCS models will have the same structure.

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Appendix

Coefficients of the generator

$$a_{21} = \left[\frac{\omega_b}{2H}\frac{X''_d - \ell_s}{X'_d - \ell_s}\frac{X_{md}}{X_f}\right], \quad a_{22} = \left[-\frac{\omega_b}{2H}\frac{X''_q - \ell_s}{X'_q - \ell_s}\frac{X_{mq}}{X_g}\right],$$

$$a_{23} = \left[\frac{\omega_b}{2H}\frac{X'_d - X''_d}{X'_d - \ell_s}\right], \quad a_{24} = \left[-\frac{\omega_b}{2H}\frac{X'_q - X''_q}{X'_q - \ell_s}\right],$$

$$a_{25} = \left[\frac{\omega_b}{2H}\left(X''_q - X''_d\right)\right],$$

$$a_{31} = \left[-\frac{1}{T'_{do}}\left(1 + \frac{(X_d - X'_d)(X'_d - X''_d)}{(X'_d - \ell_s)^2}\right)\right],$$

$${}_{32} = \left[\frac{1}{T'_{do}}\frac{(X_d - X'_d)(X'_d - X''_d)}{(X'_d - \ell_s)^2}\frac{X_f}{X_{md}}\right],$$

$$a_{33} = \left[\frac{1}{T'_{do}}\left(\frac{(X_d - X'_d)(X'_d - X''_d)}{(X'_d - \ell_s)} - (X_d - X'_d)\right)\frac{X_f}{X_{md}}\right],$$

$$b_3 = \omega_b, \quad \bar{a}_{31} = T'_{do}a_{31},$$

$$\bar{a}_{32} = T'_{do}a_{32}, \quad \bar{a}_{33} = T'_{do}a_{33}, \quad \bar{b}_{11} = T'_{qo}b_{11},$$

$$b_{11} = \left[-\frac{1}{T'_{qo}}\left(1 + \frac{\left(X_q - X'_q\right)\left(X'_q - X''_q\right)}{\left(X'_q - \ell_s\right)^2}\right)\right],$$

$$b_{12} = \left[\frac{1}{T_{qo}}\frac{\left(X_q - X'_q\right)\left(X'_q - X''_q\right)}{\left(X'_q - \ell_s\right)^2}\frac{X_g}{X_{mq}}\right],$$

$$\bar{b}_{12} = T'_{qo}b_{12}, \quad \bar{b}_{13} = T'_{qo}b_{13}, \quad \bar{b}_{21} = T''_{do}b_{21},$$

$$b_{13} = \left[\frac{1}{T'_{qo}}\left(\frac{\left(X_q - X'_q\right)\left(X'_q - X''_q\right)}{\left(X'_q - \ell_s\right)} - \left(X_q - X'_q\right)\right)\frac{X_q}{X_{mq}}\right],$$

$$b_{21} = \frac{1}{T''_{do}}\frac{X_{md}}{X_f}, \quad b_{22} = -\frac{1}{T''_{do}},$$

$$b_{23} = -\frac{1}{T''_{do}}(X'_d - \ell_s), \quad b_{31} = \frac{1}{T''_{qo}}\frac{X_{mq}}{X_g}, \quad b_{32} = -\frac{1}{T''_{qo}},$$

$$b_{33} = -\frac{1}{T''_{qo}}\left(X'_q - \ell_s\right),$$

$$c_{11} = \left[\frac{\left(X''_q - \ell_s\right)X_{mq}}{\left(X'_q - \ell_s\right)X_g\omega_b}\right], \quad c_{12} = \left[\frac{\left(X'_q - X''_q\right)}{\left(X'_q - \ell_s\right)\omega_b}\right],$$

$$c_{13} = \left[-\frac{X''_q}{\omega_b}\right], \quad c_{14} = r_s, \quad c_{15} = V^{\infty},$$

$$c_{21} = \left[\frac{(X''_d - \ell_s)X_{md}}{(X'_d - \ell_s)X_f\omega_b}\right], \quad c_{22} = \left[-\frac{\left(X'_d - X''_q\right)}{(X'_d - \ell_s)\omega_b}\right],$$

$$c_{23} = \left[\frac{X''_d}{\omega_b}\right], \quad c_{24} = r_s, \quad c_{25} = V^{\infty},$$

$$\bar{b}_{22} = -1, \quad \bar{b}_{23} = T''_{do}b_{23}, \quad \bar{b}_{31} = T''_{qo}b_{31}, \quad \bar{b}_{33} = -1,$$

$$\bar{b}_{33} = T''_{qo}b_{33}.$$

For the reduced order model

$$A_R^{-1} = \begin{bmatrix} ar_{11} & ar_{12} \\ ar_{21} & ar_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{c_{24}}{c_{14}c_{24} - c_{13}c_{23}} & \dfrac{-c_{13}}{c_{14}c_{24} - c_{13}c_{23}} \\ \dfrac{-c_{23}}{c_{14}c_{24} - c_{13}c_{23}} & \dfrac{c_{14}}{c_{14}c_{24} - c_{13}c_{23}} \end{bmatrix},$$

$$\begin{bmatrix} h_1(\mathbf{x}_1, \mathbf{x}_2) \\ h_2(\mathbf{x}_1, \mathbf{x}_2) \end{bmatrix} = -\begin{bmatrix} ar_{11} & ar_{12} \\ ar_{21} & ar_{22} \end{bmatrix} \times \begin{bmatrix} c_{11}x_4 + c_{12}x_6 + c_{15}\sin x_1 \\ c_{21}x_3 + c_{22}x_5 + c_{25}\cos x_1 \end{bmatrix}.$$

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