

This article was downloaded by:[UNAM Ciudad Universitaria]
On: 9 April 2008
Access Details: [subscription number 789204805]
Publisher: Taylor & Francis
Informa Ltd Registered in England and Wales Registered Number: 1072954
Registered office: Mortimer House, 37-41 Mortimer Street, London W1T 3JH, UK



International Journal of Control

Publication details, including instructions for authors and subscription information:
<http://www.informaworld.com/smpp/title-content=t713393989>

Parameter identification via modified twisting algorithm

M. I. Castellanos ^{ab}, I. Boiko ^c, L. Fridman ^a

^a National Autonomous University of Mexico, México, D.F.

^b Autonomous University of Ciudad Juárez, Juárez, México

^c University of Calgary, Alberta

First Published on: 13 November 2007

To cite this Article: Castellanos, M. I., Boiko, I. and Fridman, L. (2007) 'Parameter identification via modified twisting algorithm', *International Journal of Control*, 81:5, 788 - 796

To link to this article: DOI: 10.1080/00207170701636559

URL: <http://dx.doi.org/10.1080/00207170701636559>

PLEASE SCROLL DOWN FOR ARTICLE

Full terms and conditions of use: <http://www.informaworld.com/terms-and-conditions-of-access.pdf>

This article maybe used for research, teaching and private study purposes. Any substantial or systematic reproduction, re-distribution, re-selling, loan or sub-licensing, systematic supply or distribution in any form to anyone is expressly forbidden.

The publisher does not give any warranty express or implied or make any representation that the contents will be complete or accurate or up to date. The accuracy of any instructions, formulae and drug doses should be independently verified with primary sources. The publisher shall not be liable for any loss, actions, claims, proceedings, demand or costs or damages whatsoever or howsoever caused arising directly or indirectly in connection with or arising out of the use of this material.

Parameter identification via modified twisting algorithm

M. I. CASTELLANOS*†§, I. BOIKO‡ and L. FRIDMAN†

†National Autonomous University of Mexico, UNAM 04510, México, D.F.

‡University of Calgary, 2500 University Dr. N.W. Calgary, Alberta

§Autonomous University of Ciudad Juárez, Henry Dunant 4016, Juárez, México

(Received 30 April 2007; in final form 18 August 2007)

A method for identification of any even number of parameters of the transfer function from the test of the process, which involves application of the modified twisting algorithm, is proposed. Equations for determining the unknown parameters can be written separately for the magnitude and the argument of the transfer function that simplifies the task of the identification. As a result, the problem can be reduced to the iterative solution of a system of algebraic equations.

1. Introduction

Antecedents. PID control is the main type of control extensively used in various industrial applications. PID controllers are usually implemented as configurable software modules within the distributed control systems (DCS). The DCS configuration software is constantly evolving giving to the developers many new features. One of most useful features would be the controller autotuning feature. This trend can be seen in the development of new releases of such popular DCS software as Honeywell Experion PKS® and Emerson DeltaV®. The autotuning feature heavily relies on identification algorithms. Nowadays the use of first-order plus dead time or other low order underlying models may not be sufficient. Moreover, there is a strong demand in identification methods that use higher-order underlying models. One of the most convenient tests on the process in terms of the simplicity and accuracy of the identification is the relay feedback test proposed in Astrom and Hagglund (1984). This method has received a lot of attention from the worldwide research community and the industry since then. In comparison with the Astrom–Hagglund’s approach, which was aimed at obtaining the values of the ultimate gain and ultimate frequency for the PID tuning in accordance with the Ziegler–Nichols rules (Ziegler and Nichols 1942), in Luyben (1987) it was

proposed to use the relay feedback test for the process parameters identification. This idea was further developed and extended to various models and types of processes. In Kaya and Atherton (1999), for example, it was proposed to use the amplitude of the oscillations in addition to the imaginary part of the Tsytkin’s locus (Tsytkin 1984). That resulted in a precise model for two simple transfer functions. In Kaya and Atherton (1998, 2001), it was shown how the parameters of the first and second order process transfer functions with time delay could be found exactly via the use of a locus from the measurements of the asymmetric limit cycle. In Majhi and Atherton (1999), exact parameters of the first and second-order plus dead time models were obtained from measurements of the asymmetric limit cycle. In Majhi *et al.* (2001), a relay feedback and wavelet based method for estimation of unknown processes was proposed. In Boiko (2006), a method of identification of the first-order plus dead time model from a single relay feedback test, which is based on the locus of a perturbed relay system (LPRS) method (Boiko 2005), was proposed. The survey of available tuning methods and techniques based on the relay feedback test is presented in Astrom and Hagglund (1984). The identification methods use both the describing function (DF) (Atherton 1975) model and exact models of the oscillations in the relay feedback system. However, despite the obvious success of the relay feedback test identification, it is known that it offers finding only two model parameters from

*Corresponding author. Email: mcastell@uacj.mx

a single test. To identify a higher number of parameters a few tests are needed, which can be done by introducing and varying hysteresis of the relay. However, it is hard to predict what the value of the hysteresis should be to make the test efficient. In that respect, the use of the parameter that reflects the angle (phase) of the particular point of the Nyquist plot would be more convenient.

Methodology. As it is well known that the presence of unmodelled dynamics causes chattering in relay control systems see Atherton (1975), Tsytkin (1984), Boiko (2005) and Shtessel and Lee (1996), for example. Recently, it was shown (Boiko *et al.* 2004) that the second order sliding mode algorithms (Levant 1993, Bartolini *et al.* 1999), originally designed for chattering elimination, demonstrate similar oscillatory properties. But the analysis of generalized sub-optimal second order algorithm in the frequency domain shows that it is possible to adjust chattering parameters (Pisano and Usai 2003) changing the controller parameters.

In the present paper we will modify the second-order sliding mode twisting algorithm (Levant 1993) in order to make process parameter identification in a wider range using DF and LPRS methods.

Contribution. A method for identification of any even number of parameters of the transfer function from the test of the process, which involves the application on a modified twisting algorithm, is proposed. The twisting algorithm is modified in order to be able to generate the oscillations in a wide range of frequencies and amplitudes of the process transfer function. It allows one to calculate two parameters of the process from each test. It is shown that equations for determining the unknown parameters can be written separately for the magnitude and the argument of the transfer function that simplifies the task of the identification. As a result, the problem can be reduced to the iterative solution of a system of algebraic equations.

Structure of the paper. At first the idea of modification of the twisting algorithm via its DF analysis is suggested. Then the LPRS method is considered as a tool for finding the exact values of parameter of periodic oscillation. After that a methodology of identification is presented. And finally, simulation and experimental tests illustrating obtained results are presented.

2. Idea of the twisting algorithm modification

Consider the control system

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx, \end{cases} \quad (1)$$

where A and B are matrices of corresponding dimensions, $x \in R^n$, $u \in R^1$ and $y \in R^1$ can be treated as the output of the plant. We shall also use the plant description in the form of a transfer function $W(s)$, which can be obtained from (1) as follows:

$$W(s) = C(sI - A)^{-1}B. \quad (2)$$

Consider the control algorithm

$$u(t) = -c_1 \text{sign}(y) - c_2 \text{sign}(\dot{y}), \quad c_1 > 0. \quad (3)$$

This algorithm with $c_1 > c_2 > 0$ was proposed in Levant (1993). Let us relax the constraint on the sign of c_2 in comparison with the original formulation and name algorithm (3) the modified twisting algorithm (MTA); see figure 1. This allows us to increase the range of generated frequencies to the second and third quadrant of the Nyquist plot of the plant.

To explain the idea of the proposed method let us apply the DF analysis to the system (2), (3). Replace the relay functions with the DF, which for the first relay is (DF for ideal relay)

$$N_1 = \frac{4c_1}{\pi A_1}, \quad (4)$$

where A_1 is the amplitude of y ; and for the second relay

$$N_2 = \frac{4c_2}{\pi A_2}, \quad (5)$$

where A_2 is the amplitude of $\dot{y}$. Also, take into account the relationship between y and $\dot{y}$ in the Laplace domain, which gives the relationship between the amplitudes A_1 and A_2 : $A_2 = A_1 \cdot \Omega$ where Ω is the frequency of the oscillation. As a result, the DF of the twisting algorithm can be given by the following formula:

$$N = N_1 + j\Omega N_2 = \frac{4c_1}{\pi A_1} + j\Omega \frac{4c_2}{\pi A_2} = \frac{4}{\pi A_1} (c_1 + jc_2). \quad (6)$$

Let us note that the DF of the MTA does not contain the frequency and depends only on the amplitude value.

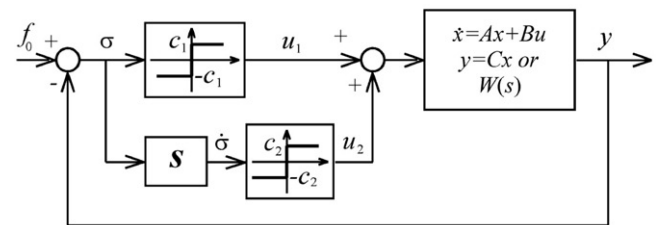


Figure 1. Diagram of twisting algorithm.

This suggests the technique of finding the parameters of the limit cycle via the solution of the complex equation

$$-\frac{1}{N(A_1)} = W(j\Omega), \quad (7)$$

where the function at the left-hand side is given by

$$-\frac{1}{N} = \frac{\pi A_1}{4(c_1^2 + c_2^2)} \cdot (-c_1 + jc_2). \quad (8)$$

The graphical illustration of this technique for the solution of the equation (7) is given in figure 2. The function $-1/N$ is a straight line the slope of which depends on the c_2/c_1 ratio. It is located in the second or third quadrant of the complex plane. The point of the intersection of this function and of the Nyquist plot $W(j\omega)$ provides the solution of the periodic problem. This point gives the frequency of the oscillation Ω and the amplitude A_1 .

The angle of $\angle W(j\omega)$ is defined by equation (9)

$$\angle W(j\omega) = \angle -\frac{1}{N} = -\pi - \arctan \frac{c_2}{c_1}. \quad (9)$$

From the figure 2 and equations (7), (8) it is easy to conclude that the twisting algorithm in its original form (with $c_1 > c_2 > 0$), allows only the values of the frequency corresponding to the Nyquist plot in the domain $-5/4\pi < \angle W(j\omega) < -\pi$. The use of the MTA provide the possibility of utilization of all frequencies of the Nyquist plot of the plant corresponding to the angle $-3/2\pi < \angle W(j\omega) < -\pi/2$. Furthermore, if we need to identify the points Nyquist plot at lower frequencies an integrator can be introduced in series with the MTA. That would allow one to excite oscillation in the controller-plant loop at frequencies from the 4th quadrant of the plant Nyquist plot (this can also be considered as clockwise rotation of the Nyquist plot by $\pi/2$ rad). However, the resolution of the method decreases as the generated frequency of

the oscillations approaches zero—due to low influence of higher time constants on the frequency in this case.

The DF analysis provides a very demonstrative proof of the possible existence of a periodic solution in the system with the MTA. The DF method is an approximate one and more exact values of parameters would be desirable. This can be provided using the LPRS method (Boiko 2005).

3. LPRS analysis of MTA

The LPRS can provide an exact solution of the periodic problem in a relay feedback system having a plant (1) and the control being the hysteretic relay function

$$u = \begin{cases} c & \text{if } \sigma \geq b \text{ or } (\sigma > -b \text{ and } \dot{\sigma} < 0) \\ -c & \text{if } \sigma \leq -b \text{ or } (\sigma < b \text{ and } \dot{\sigma} > 0), \end{cases} \quad (10)$$

where σ is the error signal ($\sigma = f_0 - y$), f_0 is the system input (subscript “0” denotes the constant input). The LPRS for such a system is defined as follows (Boiko 2005):

$$J(\omega) = -\frac{1}{2} \lim_{f_0 \rightarrow 0} \frac{\sigma_0}{u_0} + j \frac{\pi}{4c} \lim_{f_0 \rightarrow 0} y(t) \Big|_{t=0}. \quad (11)$$

where $t=0$ is the time of the switch of the relay from “ $-c$ ” to “ $+c$ ”, ω is the frequency of the self-excited oscillations varied by changing the hysteresis $2b$ while all other parameters of the system are considered constant. σ_0 , u_0 are average (over the period of the oscillations) values of the error signal and of the control respectively σ_0 , u_0 and $y(t)|_{t=0}$ are, therefore, functions of ω , u . Thus, $J(\omega)$ specifies the response of the linear plant to its non-symmetric pulse waveform input $u(t)$ subject to $f_0 \rightarrow 0$ as the frequency ω is varied. The real part of $J(\omega)$ contains the information about the equivalent gain of the relay, and the imaginary part of $J(\omega)$ comprises the condition of the switching of the relay and, consequently, contains information about the frequency of the oscillations. It is worth mentioning that although the LPRS is defined via the parameters of the oscillations, it is a function of the plant parameters only. A few techniques of the LPRS computing were developed. One of them (that will be used further in this paper) offers computing of the LPRS as a series of the real and imaginary parts of the plant transfer function

$$J(\omega) = \sum_{k=1}^{\infty} (-1)^{k+1} \operatorname{Re} W(k\omega) + j \sum_{k=1}^{\infty} \operatorname{Im} W[(2k-1)\omega]/(2k-1). \quad (12)$$

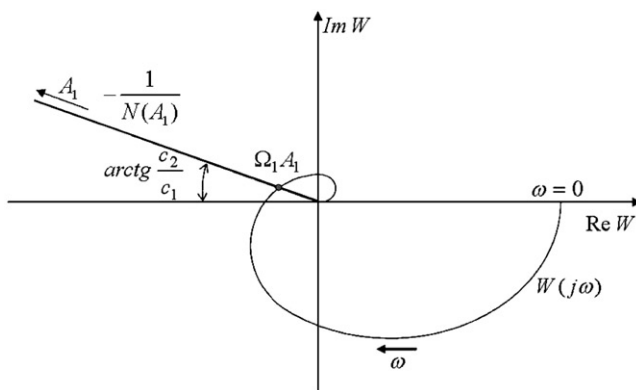


Figure 2. Finding the periodic solution.

With the LPRS computed analysis of periodic motions in a relay feedback system becomes an easy task. The frequency of the periodic motion is found from the following equation:

$$\text{Im}(\Omega) = -\frac{\pi b}{4c}. \quad (13)$$

However, to be able to use this method for the MTA, we need to transform the original problem into an equivalent one that can be solved with the use of the LPRS. Let us take a closer look at figure 1. We can see that the functions of the two relays are different. The function of the first relay is to generate a regular asymptotic sliding mode control like every relay feedback system. The function of the second relay in the frequency domain can be described as providing a phase lead by injecting a control that depends on the derivative of the output variable. Moreover, c_1 is always greater than c_2 . This enables us to consider that the first relay is the main one and the second relay is of secondary function and to transform the original diagram figure 1 into the following structure figure 3.

In figure 3 we are going to treat the part of the system denoted by the dashed line as a new plant of the relay system (the equivalent plant). This structure completely complies with the relay feedback system structure. The equivalent plant, however, is nonlinear with the nonlinear function being the second relay. For that reason, the LPRS method needs to be modified to accommodate the nonlinearity of the plant. Let us begin from considering the imaginary part of the LPRS.

The imaginary part of the LPRS is the value of the output of the plant at the instant of the relay switch from “−” to “+” (with a coefficient). This follows from the LPRS definition (11). Therefore, the imaginary part of the LPRS can be found as the output of the plant at the switching instant if a periodic square-wave pulse signal u_1 is applied to the plant input. Note also that the signal u_2 is also applied to the plant input and the output of the plant y can be considered as a sum of two outputs y_1 and y_2 , each of them is a response to the control u_1 and u_2 respectively. Moreover, y_2 can be obtained by time shifting and scaling of y_1 . Introduce the following function that can be helpful for the LPRS computing. Let $L(\omega, \theta)$ be the function that denotes a linear plant output (with a coefficient) at the instant $t = \theta T$ (with T being half a period) if a periodic square-wave pulse signal of amplitude c is applied to the plant:

$$L(\omega, \theta) = \frac{\pi}{4c} y(t) \Big|_{t=\theta(2\pi/\omega)} \quad (14)$$

where $\theta \in [-1, +1]$, $\omega \in [0, \infty]$. Positive values of θ correspond to the time following the switching instant, negative values to the time preceding the switching

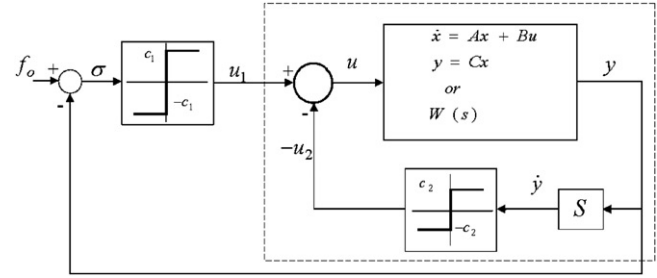


Figure 3. Transformed system with twisting algorithm.

instant. If we compare this formula with the LPRS definition we would find that: $\text{Im}J(\omega) = L(\omega, 0)$. Analysis of the Fourier series of a linear plant output leads to the following expression for $L(\omega, \theta)$:

$$L(\omega, \theta) = \sum_{k=1}^{\infty} \frac{1}{2k-1} \{ \sin[(2k-1)2\pi\theta] \cdot \text{Re}W[(2k-1)\omega] + \cos[(2k-1)2\pi\theta] \cdot \text{Im}W[(2k-1)\omega] \}. \quad (15)$$

With the formula of $L(\omega, \theta)$ available we can easily write an expression for $\text{Im}J(\omega)$ of our system

$$\text{Im}J(\omega) = L(\omega, 0) + \frac{c_2}{c_1} L(\omega, \theta). \quad (16)$$

In formula (16), the value of time shift θ between the switching of the first and second relays is unknown. It can be found from the following equation:

$$y(\theta) = 0, \quad (17)$$

which can be expressed via the function $L(\omega, \theta)$ as follows:

$$c_1 L_1(\omega, -\theta) + c_2 L_1(\omega, 0) = 0. \quad (18)$$

In (18), L_1 is the function $L(\omega, \theta)$ for which the transfer function in formula (15) is $W_1(s) = sW(s)$ (transfer function from the control to dy/dt). Therefore, the methodology of analysis of the periodic motions in the system with the twisting algorithm is as follows. At each frequency point of the LPRS, equation (18) is solved for the time shift θ (in parts of the period) between the switches of the two relays, where function $L(\omega, \theta)$ is computed as per (15). After that the imaginary part of the LPRS is computed as per (16). With the imaginary part available, the frequency of the oscillations is found from equation (13) with $c = c_1$.

We shall now develop a technique for computing the real part of the LPRS. Although the real part is not explicitly used in our analysis it would be useful to see how the addition of the second relay changes the location of the LPRS on the complex plane. Assume that the plant transfer function does not contain integrators (poles with zero real part).

Formula (12) consists of two converging series in which every next term adds accuracy to the LPRS computing. Usually the convergence at high frequencies (close to the frequency of the periodic solution) is very fast and even the first term provides a relatively high accuracy. Using only the first terms in (12) would be equivalent to the DF method formula.

For the objectives stated in this paper the use of only first term of (12) for the real part computing would be a reasonable approach. We should note also that the frequency of the oscillations is computed exactly. Therefore, the real part of the LPRS can be computed as follows:

$$ReJ(\omega) \approx Re \left[\frac{W(j\omega)}{1 + j\omega \cdot q_2(A_2) \cdot W(j\omega)} \right], \quad (19)$$

where $q_2 = 4c_2/\pi A_2$, $A_2 = (4c_1/\pi)\omega |W(j\omega)|$.

4. Parameter identification via MTA

Varying c_1 and c_2 we can change the parameters of the oscillations. Each combination of c_1 and c_2 provides a certain amplitude and frequency of a periodic motion. Therefore, n tests are needed to identify $2n$ process parameters.

A possible way of choosing the relay amplitudes is to use c_1/c_2 ratio of the same magnitude and different signs for two different tests. This would allow us to identify the plant (process) at the phase characteristic equally distances from $-\pi$ rad (symmetric with respect to the real axis).

For example if we need to identify six parameters we can select the values of $c_2/c_1 = 1/2$, 0 and $-1/2$ corresponding to the angles $-7\pi/6$ rad, $-\pi$ rad, and $-5\pi/6$ rad.

Suppose that each test provides the amplitude A_i and the frequency Ω_i ($i = 1, \dots, n$) of the oscillations. Assume that the underlying model of the process is given in the form of the following transfer function (which provides a relatively universal model if allow for complex values of the time constants):

$$W(s) = \frac{K(\Upsilon_1 s + 1) \cdots (\Upsilon_m s + 1)}{(T_1 s + 1) \cdots (T_k s + 1)}, \quad (20)$$

where $m < k$, $m + k = 2n - 1$. It is a function with k poles, m zeros and gain K . From the describing function (7), rewriting equations (8) and (7) we obtain the equation of harmonic balance for each test point: $W(j\Omega_i)N(A_i) = -1$. Rewriting this equation into the format more convenient for the solution, we obtain the following set of equations:

$$W(j\Omega_i) = -N(A_i)^{-1}. \quad (21)$$

Taking the magnitude and the argument yields

$$|W(j\Omega_i)| = |-N(A_i)^{-1}| = \frac{A_1 \pi}{4\sqrt{c_{1i}^2 + c_{2i}^2}} \quad (22)$$

$$\angle W(j\Omega_i) = -\pi - \arctan\left(\frac{c_{2i}}{c_{1i}}\right). \quad (23)$$

Further, taking the logarithms of both parts of the equation for the magnitudes leads to the following equation:

$$\ln |W(j\Omega_i)| = \ln \left| \frac{K(\Upsilon_1 i\Omega_i + 1) \cdots (\Upsilon_m i\Omega_i + 1)}{(T_1 i\Omega_i + 1) \cdots (T_k i\Omega_i + 1)} \right| \quad (24)$$

which simplifies to

$$\begin{aligned} \ln |W(j\Omega_i)| &= \ln K + \sum_{l=1}^m \ln \sqrt{\Upsilon_l^2 \Omega_i^2 + 1} - \sum_{l=1}^k \ln \sqrt{T_l^2 \Omega_i^2 + 1} \\ &= \ln \frac{A_1 \pi}{4\sqrt{c_{1i}^2 + c_{2i}^2}} \end{aligned} \quad (25)$$

and yields

$$\begin{aligned} \ln K + \frac{1}{2} \sum_{l=1}^m \ln(\Upsilon_l^2 \Omega_i^2 + 1) - \frac{1}{2} \sum_{l=1}^k \ln(T_l^2 \Omega_i^2 + 1) \\ = \ln \frac{1}{4} \frac{A_1 \pi}{\sqrt{c_{1i}^2 + c_{2i}^2}}. \end{aligned} \quad (26)$$

In the same way, we can write for the arguments of (21)

$$\sum_{l=1}^m \phi(\Upsilon_l j\Omega_i + 1) - \sum_{l=1}^k \phi(T_l j\Omega_i + 1) = -\pi - \arctan\left(\frac{c_{2i}}{c_{1i}}\right). \quad (27)$$

Evaluating the angle for each test point, and simplifying we obtain

$$\sum_{l=1}^m \arctan \Upsilon_l \Omega_i - \sum_{l=1}^k \arctan T_l \Omega_i = -\pi - \arctan\left(\frac{c_{2i}}{c_{1i}}\right). \quad (28)$$

Solving the set of equations (26) and (28) obtain the unknown parameters $K, \Upsilon_1, \dots, \Upsilon_m, T_1, \dots, T_k$.

It is known that the DF technique does not provide exact results. The LPRS technique can be used to obtain an exact solution.

Once we measure the amplitude and frequency of the oscillations at each test point, we can solve the equation for the imaginary part (16) and the equation for the shift between the switches of the relays (17) and find the parameters of the plant (20).

For each test point i , and known c_{2i} and c_{1i} the following equation has to be solved:

$$L(\Omega_i, 0) + \frac{c_{2i}}{c_{1i}} L(\Omega_i, \theta_i) = 0. \quad (29)$$

Substituting (15) in (16) we have

$$\begin{aligned} & \sum_{k=1}^{\infty} \frac{1}{2k-1} \cdot \operatorname{Im} W[(2k-1)\Omega_i] \\ & + \frac{c_{2i}}{c_{1i}} \sum_{k=1}^{\infty} \frac{1}{2k-1} \{ \sin[(2k-1)2\pi\theta_i] \cdot \operatorname{Re} W[(2k-1)\Omega_i] \\ & + \cos[(2k-1)2\pi\theta_i] \cdot \operatorname{Im} W[(2k-1)\Omega_i] \} = 0. \end{aligned} \quad (30)$$

For each test we will have a system of equations that allow us to find the exact plant parameters. The set of equations for the DF identification (26) and (28) or LPRS identification equations (30) and (19) can be solved by means of the Newton–Raphson method:

$$x_{j+1} = x_j - \alpha \cdot J(f(x_j))^{-1} \cdot f(x_j), \quad (31)$$

where $f(K, \Upsilon_1, \dots, \Upsilon_m, T_1, \dots, T_k) = 0$, $J(f(x_j))$, is the Jacobian of f , at the points $x_j = [(K_j, \Upsilon_{1j}, \dots, \Upsilon_{mj}, T_{1j}, \dots, T_{kj})]^T$, α is parameter determine the speed of convergence. The proposed algorithm converges, if $\det|J(f(x_j))| \neq 0$. It can be noted that there may exist some nonminimum phase or unstable plants that would provide multiple intersections of the line $-N^{-1}(a)$ by the Nyquist plot of the plant. This potentially might result in using a wrong periodic solution and obtaining wrong parameters of the plant. Should this situation be possible (which follows from the plant model) a few different initial points for solving the equations should be used to make sure that the solution matches to the generated periodic motion. However, we do not consider this kind of situation and assume the uniqueness of the Nyquist plot with respect to a radial direction (uniqueness of the phase characteristic).

5. Examples

Example 1: Identification parameters of an inverted pendulum.

The advantage of using the MTA is the possibility of ensuring the intersection of the Nyquist plot of the process transfer function by the negative reciprocal of the describing function in the second and third quadrants of the complex plane (but not only on the real axis as in the case of the relay feedback test). The pendulum parameters are as follows: $M=0.5$ kg is the mass of the cart, $m=0.2$ kg is the mass of the pendulum, $b=0.1$ $N \cdot m^{-1} \cdot s^{-1}$ is the friction of the cart, $l=0.3$ m is the length of the pendulum (between the center of mass and

the axel), $I=0.006$ $kg \cdot m^2$ is the inertia of the pendulum, F is the force applied to the cart, x is the cart position, θ is the pendulum angle from vertical. The linearized system of equations can be represented in state-space form as follows:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{(I+ml^2)b}{(I(M+m)+Mml^2)} & \frac{(m^2gl^2)}{(I(M+m)+Mml^2)} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{mlb}{(I(M+m)+Mml^2)} & \frac{mgl(M+m)}{(I(M+m)+Mml^2)} & 0 \end{bmatrix} \quad (32)$$

$$B = \begin{bmatrix} 0 \\ i+ml^2 \\ \frac{I(M+m)+Mml^2}{0} \\ \frac{ml^2}{I(M+m)+Mml^2} \end{bmatrix} \quad (33)$$

$$C = [0 \quad 0 \quad 1 \quad 0].$$

Therefore, the plant transfer function can be obtained as follows:

$$\begin{aligned} P &= (M+m)(I+ml^2) - (ml)^2 \\ \frac{\theta(s)}{U(s)} &= \frac{(ml/P)s}{s^3 + (b(I+ml^2)/P)s^2 - ((M+m)mgl/P)s - (bmgl/P)} \end{aligned} \quad (34)$$

which yields (considering the given parameter values):

$$\frac{\theta(s)}{U(s)} = \frac{4.5455s}{s^3 + 0.1818s^2 - 31.1818s - 4.4545} \quad (35)$$

or in the form containing the time constants:

$$\frac{\theta(s)}{U(s)} = -\frac{1.0204s}{(-0.1858s+1)(0.17844s+1)(7.0013s+1)}. \quad (36)$$

Run the test for two different combinations of c_1 and c_2 . If we analyse the system with plant (36) by means of the DF and the LPRS the values of the frequency and the amplitude would be as given in tables 1 and 2.

Table 1. The DF analysis.

Test point			Frequency (rad/s)	Amplitude
i	c_1	c_2	Ω_i	A
1	5	-1	0.717	0.1406
2	10	-1	1.453	0.1359

Replacing the frequency and the amplitude in (26) and (28) for the first test point we have (with $c_1=5$, $c_2=-1$)

$$\begin{aligned} & \frac{1}{2} \ln(0.9884^2 \cdot \Upsilon_l^2) - \frac{1}{2} \sum_{l=1}^3 \ln(0.9884^2 \cdot T_l^2 + 1) \\ &= \ln \frac{\pi \cdot 0.1408}{4\sqrt{5^2 + 1^2}} \end{aligned} \quad (37)$$

$$-\frac{\pi}{2} - \sum_{l=1}^3 \arctan(0.9884 \cdot T_l) + \pi + \arctan\left(\frac{-1}{5}\right) = 0 \quad (38)$$

and for the second test point ($c_1=10$, $c_2=-1$).

$$\begin{aligned} & \frac{1}{2} \ln(1.8162^2 \cdot \Upsilon_l^2) - \frac{1}{2} \sum_{l=1}^3 \ln(1.8162^2 \cdot T_l^2 + 1) \\ &= \ln \frac{\pi \cdot 0.1316}{4\sqrt{10^2 + 1^2}} \end{aligned} \quad (39)$$

$$\frac{\pi}{2} - \sum_{l=1}^3 \arctan(1.8162 \cdot T_l) + \pi + \arctan\left(\frac{-1}{10}\right) = 0. \quad (40)$$

The parameters (table 3) obtained via solving the DF of equations (37)–(40), are as follows: $\Upsilon_1=-0.7122$, $T_1=-0.1858$, $T_2=0.1920$, $T_3=4.823$, therefore, the transfer function of the identified plant is (figure 4)

$$W_{DF1}(s) = \frac{-0.7122s}{(-0.1858s + 1)(0.192s + 1)(4.823s + 1)} \quad (41)$$

and the parameters obtained via solving the LPRS equations are $\Upsilon_1=-1.0204$, $T_1=-0.1796$,

Table 2. The LPRS analysis.

Test point			Frequency (rad/s)	Amplitude
i	c_1	c_2	Ω_i	A
1	5	-1	0.9884	0.1408
2	10	-1	1.8162	0.1316

Table 3. The parameters identified.

	Parameters			
	Υ_1	T_1	T_2	T_3
Original parameters	-1.0204	-0.1858	0.1784	7.0013
DF identification	-0.7122	-0.1858	0.1920	4.823
LPRS identification	-1.0204	-0.1796	0.1796	7.001

$T_2=0.1796$, $T_3=7.001$. The transfer function of the identified plant is

$$W_{LPRS1}(s) = \frac{-1.0204s}{(-0.1796s + 1)(0.1796s + 1)(7.001s + 1)}. \quad (42)$$

Example 2: Experimental identification parameters of the RC third-order low-pass filter

The circuit of the RC third-order low-pass filter with decoupling amplifiers, can be seen in the figure 5. The experimental setup is implemented with operational amplifiers LM741 of Texas Instruments and ceramic capacitors, the bipolar source of variable voltage is a programmable power supply HM 8142, in order to verify the measurements of the frequency and amplitude of the control and the output signal in the circuit we used an oscilloscope Agilent 54621A. The experimental setup includes a PC equipped with an Dspace1103 data control and acquisition card. The controller was implemented using Matlab and Dspace programming language allowing debugging, virtual oscilloscope, automation functions, and data storage during the experiments. The sampling frequency for control implementation has been set to 10 kHz.

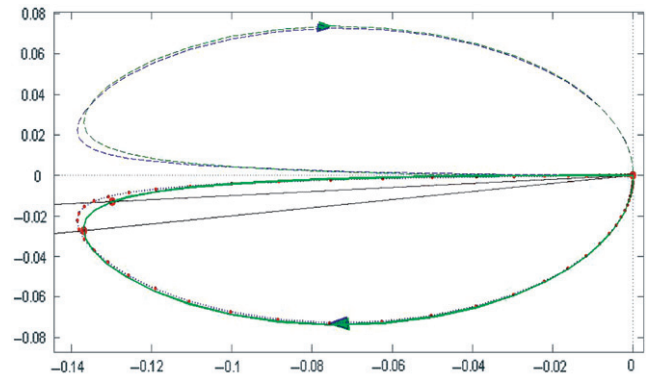


Figure 4. Nyquist plot for inverted pendulum transfer function.

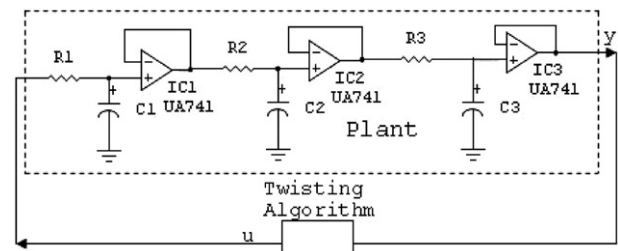


Figure 5. RC third-order low-pass filter with decoupling amplifiers.

Table 4. Experimental data.

Test point			Frequency (rad/s)	Amplitude
i	c_1	c_2	Ω_i	A
1	5	0	125.72	0.78
2	5	-1	113.09	0.96

Table 5. The parameters identified.

	Parameters			
	K	T_1	T_2	T_3
Original parameters	1.0204	0.0096	0.0136	0.0194
DF Identification	1.0430	0.0108	0.0108	0.0228
LPRS Identification	1.0204	0.0104	0.0104	0.0238

The filter transfer function is as follows:

$$W_{RCLP}(s) = \frac{1}{(R_1 \cdot C_1 \cdot s + 1)(R_2 \cdot C_2 \cdot s + 1)(R_3 \cdot C_3 \cdot s + 1)}. \quad (43)$$

The measured values of R and C are as follows: $R_1 = 9.947 \text{ K}\Omega$, $R_2 = 9.886 \text{ K}\Omega$, $R_3 = 9.887 \text{ K}\Omega$, $C_1 = 0.968 \text{ }\mu\text{F}$, $C_2 = 1.381 \text{ }\mu\text{F}$ and $C_3 = 1.968 \text{ }\mu\text{F}$, so the parameters $K = 1.0204$, $T_1 = 0.0096$, $T_2 = 0.0136$ and $T_3 = 0.0194$. Therefore, the actual transfer function is

$$W_{REAL}(s) = \frac{1}{(0.0096 \cdot s + 1)(0.0136 \cdot s + 1)(0.0194 \cdot s + 1)}. \quad (44)$$

Considering two test points, in which the values of c_1 and c_2 are 5 and 0 respectively for the first test and $c_1 = 5$ and $c_2 = -1$ for the second test, the experimental frequencies shown in table 4 and amplitudes were obtained.

The parameters (table 5) identified via the DF model are $K = 1.043$, $T_1 = 0.0108$, $T_2 = 0.0108$, $T_3 = 0.0228$, therefore, the transfer function of the identified plant is

$$W_{DF}(s) = \frac{1.043}{(0.0108 \cdot s + 1)(0.0108 \cdot s + 1)(0.0228 \cdot s + 1)} \quad (45)$$

and the parameters found via the LPRS based identification are $K = 1.0204$, $T_1 = 0.0104$, $T_2 = 0.0104$ and $T_3 = 0.0238$.

$$W_{LPRS}(s) = \frac{1.0204}{(0.0104 \cdot s + 1)(0.0104 \cdot s + 1)(0.0238 \cdot s + 1)} \quad (46)$$

Table 6. Experimental data.

Test point		Frequency (rad/s)	Amplitude
c_1	c_2	Ω_i	A
5	-2.5	96.8	1.45

Table 7. DF data.

Test point		Frequency (rad/s)	Amplitude
c_1	c_2	Ω_i	A
5	-2.5	91	1.63

Table 8. LPRS data.

Test point		Frequency (rad/s)	Amplitude
c_1	c_2	Ω_i	A
5	-2.5	91.8	1.42

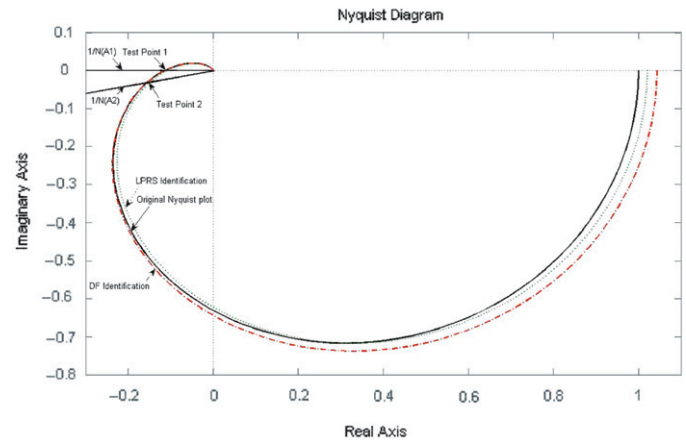


Figure 6. Nyquist plots of transfer functions of RC third-order low-pass filter

Now analyse the parameters of the oscillations in the system having the plant transfer function (obtained via the DF and the LPRS models) and the twisting algorithm in the test point $c_1 = 5$ and $c_2 = -2.5$ (see tables 6–8 and figure 6).

6. Conclusions

A method for identification of any even number of parameters of the transfer function from the test on the process is proposed. With this aim the modification of the twisting algorithm is proposed allowing to enlarge the bandwidth of frequencies which could be used for the identification. Each test allows one to define two algebraic equations for the process transfer function parameters. The equations for determining the unknown parameters can be written separately for the magnitude and the argument of the transfer function that simplifies the task of the identification. As a result, the problem can be reduced to the iterative solution of a system of algebraic equations.

The advantage of the proposed approach is the possibility of the excitation of the oscillations at a given argument (phase lag) of the process transfer function. It is demonstrated that the proposed technique is feasible and provides a satisfactory accuracy of identification.

References

- K.I. Astrom and T. Hagglund, "Automatic tuning of simple regulators with specifications on phase and amplitude margins", *Automatica*, 20, pp. 645–651, 1984.
- D.P. Atherton, *Nonlinear Control Engineering—Describing Function Analysis and Design*, Workingham, Berks, UK: Van Nostrand Company Limited, 1975.
- G. Bartolini, A. Ferrara, A. Levant and E. Usai, "On second order sliding mode controllers", in *Variable Structure Systems, Sliding Mode and Nonlinear Control*, K.D. Young and U. Ozguner, Eds, Berlin: Springer Verlag, 1999, pp. 329–350.
- I. Boiko, "Oscillations and transfer properties of relay servo systems – the locus of a perturbed relay system approach", *Automatica*, 41, pp. 677–683, 2005.
- I. Boiko, "Method and apparatus for tuning a PID controller", US Patent No. 7035695, 2006.
- I. Boiko, L. Fridman and M.I. Castellanos, "Analysis of second-order sliding mode algorithms in the frequency domain", *IEEE Trans. Automat. Contr.*, 49, pp. 946–950, 2004.
- I. Kaya and D.P. Atherton, "An improved parameter estimation method using limit cycle data", *UKACC Internat. Conf. Contr., IEE*, pp. 682–687, 1998.
- I. Kaya and D.P. Atherton, "A PI-PD controller design for integrating processes", in *Proc. 1999 American Control Conference*, San Diego, CA, USA, 1999, pp. 258–262.
- I. Kaya and D.P. Atherton, "Parameter estimation from relay autotuning with asymmetric limit cycle data", *J. Proc. Contr.*, 11, pp. 429–439, 2001.
- A. Levant, "Sliding order and sliding accuracy in sliding mode control", *Int. J. Contr.*, 58, pp. 1247–1263, 1993.
- W.L. Luyben, "Derivation of transfer functions for highly nonlinear distillation columns", *Ind. Eng. Chem. Res.*, 26, pp. 2490–2495, 1987.
- S. Majhi and D.P. Atherton, "Autotuning and controller design for processes with small time delays", *IEE Proc.—Contr. Theo. Appl.*, 146, pp. 415–425, 1999.
- S. Majhi, J.S. Sahmbi and D.P. Atherton, "Relay feedback and wavelet based estimation of plant model parameters", *Proceedings on 40 IEEE Conference on Decision and Control*, Florida, USA, 2001, pp. 3326–3331.
- A. Pisano and E. Usai, "An improved second-order sliding-mode control scheme robust against the measurement noise", *Proceedings on 42nd Conference on Decision and Control*, Maui, Hawaii, 2003, pp. 3519–3525.
- Y.B. Shtessel and Y.J. Lee, "New approach to chattering analysis in systems with sliding Modes", *Proceedings of the 35th Conference on Decision and Control*, Kobe, Japan, 1996, pp. 4014–4019.
- Ya.Z. Tsypkin, *Relay Control Systems*, Cambridge, England: Cambridge University Press, 1984.
- J.G. Ziegler and N.B. Nichols, "Optimum settings for automatic controllers", *Trans. Amer. Soc. Mech. Eng.*, 64, pp. 759–768, 1942.