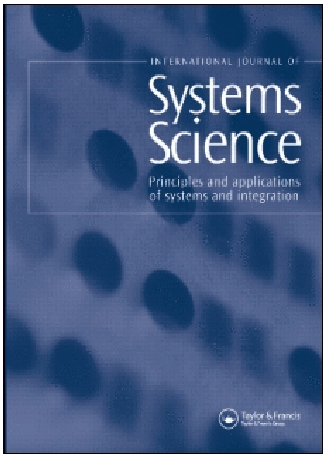

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Describing function analysis of second-order sliding mode observers

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The second-order sliding mode observer dynamics are analyzed in the frequency domain. The so-called super-twisting algorithm is utilized for generating the second-order sliding mode in the observer dynamical system. The frequency response of the observer dynamics is obtained and used as a characteristic of the observer. The analysis proposed is based on the describing function method and the concept of the equivalent gains of nonlinear functions of the super-twisting algorithm.

Keywords: Sliding mode; Observers; Frequency-domain methods; Describing function

1. Introduction

Since the introduction of the observer by Luenberger (Luenberger 1966) the problem of the observer design has become a popular area of research, which has also led to numerous industrial applications. The appealing feature of the observer is in convenience of the use and the cost effect provided by the replacement of "hard" sensors and transmitters with "soft" ones, which are implemented via computer algorithms. One of the most successful applications of the sliding mode (SM) principles is the SM observer (Utkin 1992, Edwards and Spurgeon 1998, Walcott and Zak 1998, Barbot *et al*. 2002, Edwards *et al*. 2002, Poznyak 2004). In this type of observer, the system is designed in such a way that the difference between the output of the observer and the output of the plant becomes the *sliding variable*. The discontinuous control is designed to provide the existence of SM in the observer dynamical system. Once the SM is generated, the measurable states of the observer become the estimates of the system states being observed.

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Some features of the second-order SM (SOSM) algorithms (Levant 1998, 2003, Alvarez *et al*. 2000, Bartolini *et al*. 2003, Orlov *et al*. 2003, Shtessel *et al*. 2003, Davila and Fridman 2004, Pisano and Usai 2004, Davila *et al*. 2005) make them suitable for the use in SM observers instead of the conventional relay control. These features are: the higher accuracy of SM motions, the finite-time convergence, and smaller amplitudes of chattering (Boiko *et al*. 2004, Boiko and Fridman 2005).

In the present article, analysis of dynamical properties of the SOSM observer is done in the frequency domain. In particular, the observer response to harmonic inputs of various frequencies is evaluated. The general concept of analysis is based on the describing function (DF) method (Atherton 1975). The frequency-domain analysis becomes possible due to the transformation of the original nonlinear discontinuous system into the linearized dynamics for the averaged variables – via the concept of the *equivalent gain* (Atherton 1975, Boiko 2005). The discrete-time realization of the observer is accounted for as an equivalent delay (Astrom and Wittenmark 1998, Fridman *et al*. 2004, Miloslavljevic 2004). The equivalent delay is determined via matching the frequency of chattering in the original discrete-time

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system and the equivalent continuous-time system with the delay. As a result of such analysis, the frequency domain dynamical characteristics of the observation error can be obtained. This allows for quantitative analysis of the observation accuracy at different input frequencies.

The article is organized as follows. In the first section of the article, the dynamical model of the observer and plant suitable for the frequency-domain analysis is considered, and analysis of chattering is done via application of the DF method. Then the concept of the equivalent gain and its derivation from the DF model of the system are presented. Finally an example of the frequency-domain analysis is given.

2. Second-order sliding mode observer model

Consider an n-dimensional version of the observer proposed by Utkin (Utkin 1992). Let the linear plant, the states of which are supposed to be observed, be the n-th order dynamical system:

$$\dot{\mathbf{x}} = \mathbf{A}\mathrm{x} + \mathbf{B}u \tag{1}$$

$$y = \mathbf{C}\mathrm{x}, \tag{2}$$

where $\mathbf{A} \in R^{n\times n}$, $B \in R^{n\times 1}, \mathbf{C} \in R^{1\times n}$ are matrices. The pair $(\mathbf{C}, \mathbf{A})$ is assumed to be observable.

The SOSM observer can be designed in the same form as the original system (1), (2) with an additional control input that depends on the error between the output of the observer and the output of the plant (system to be observed):

$$\dot{\widehat{\mathbf{x}}} = \mathbf{A}\widehat{x} + \mathbf{B}u + \mathbf{L}v \tag{3}$$

$$\widehat{y} = \mathbf{C}\widehat{x} \tag{4}$$

where $\widehat{\mathbf{x}}$ is an estimate of the system state vector, $\widehat{y}$ is an estimate of the system output, $\mathbf{L} \in R^{n\times 1}$ is a gain matrix, v is the output of the SOSM algorithm. For the purpose of driving $\hat{y}(t)$ to $y(t)$, let us use the so-called super-twisting algorithm (Levant 1998, 2003):

$$v = v_1(t) + v_2(t) \tag{5}$$

$$\dot{v}_1(t) = \gamma\,\mathrm{sign}\,(\sigma) \tag{6}$$

$$v_2(t) = \begin{cases} \lambda|s_0|^{\rho}\,\mathrm{sign}\,(\sigma) & \text{if } |\sigma| > s_0 \\ \lambda|\sigma|^{\rho}\,\mathrm{sign}\,(\sigma) & \text{if } |\sigma| \le s_0 \end{cases} \tag{7}$$

$$\sigma = y - \widehat{y} \tag{8}$$

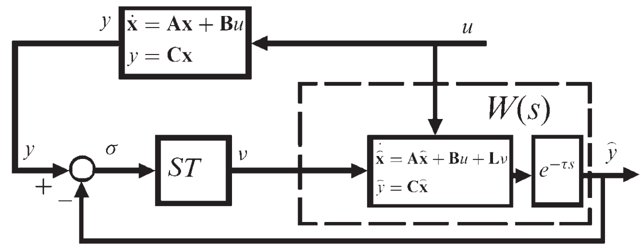

Figure 1. Plant and observer model.

where γ, ρ, λ and s_0 are design parameters, $0 < \rho \le 0.5$ (we will consider below only the case of $\rho = 0.5$), σ is the sliding variable. It can be shown that if the elements of $\mathbf{L}$ are sufficiently large the sliding variable σ converges to zero in finite time. Methods of design of the SM observers are considered in detail in (Utkin 1992, Edwards *et al.* 1998).

Let us consider the observer as a dynamical system that has two inputs and one output. One of those two inputs $y(t)$ must be followed (tracked) by the observer output $\widehat{y}(t)$ as precisely as possible. The other input $u(t)$ can be treated as a feedforward. Therefore, it makes sense to consider the observer as a *feedback–feedforward system* that utilizes the super-twisting algorithm in the controller.

In the dynamical model of the super-twisting algorithm the discrete-time realization can be accounted for as an equivalent time delay as per Astrom *et al.* (1998), Fridman *et al.* (2004), and Miloslavljevic (2004). However, the determination of the equivalent delay is a subject that is worth a separate analysis. Further, an approach based on matching the frequencies of chattering in the discrete-time and continuous-time realizations is presented. Therefore, the equivalent continuous-time SM dynamics may be given by the diagram figure 1.

3. Analysis of self-excited oscillations in SM observer dynamics

Obviously, in the steady mode, periodic motions occur in the observer loop. In the case of the first-order SM control the period of those motions would be equal to two sampling (execution) periods. In the case of the SOSM control, when the discontinuous control is applied to the second derivative of the output, there may be a few different situations. We can assume that two execution periods are required to change the sign of the output and, therefore, the period of the self-excited oscillations (chattering) would be equal to four execution periods. This assumption does not however affect the generality of the proposed approach.

On the other hand, in the continuous-time model (figure 1), by selecting the equivalent delay to zero we can obtain infinite frequency of chattering in the observer loop. Varying the delay value, we can achieve the frequency of chattering in the continuous-time model equal to the frequency of chattering in the discrete-time model. This would correspond to the equivalency of the two models. Let us refer to the corresponding time delay in the continuous-time model as the *equivalent delay*. First, assuming that the equivalent delay is known carry out analysis of the self-excited oscillation (chattering) that occurs in the observer loop of the continuous-time model due to the use of a discontinuous control algorithm. The DF of the first component of the super-twisting algorithm can be written as follows (Atherton 1975):

$$N_1 = \frac{4\gamma}{\pi a_y} \frac{1}{j\omega} \tag{9}$$

which is a result of the cascade connection of the ideal relay with the DF equal to $4\gamma/(\pi a_y)$ and the integrator with the transfer function $1/s$ (for the harmonic signal the Laplace variable s can be replaced with $j\omega$).

For the second control component having the square root nonlinearity ($\rho = 0.5$) the DF formula can be derived as (Boiko and Fridman 2005):

$$\begin{aligned} N_2 &= \frac{2}{\pi a_y} \int_0^{\pi} \lambda \sqrt{a_y \sin\psi} \sin\psi \, \mathrm{d}\psi \\ &= \frac{2\lambda\sqrt{a_y}}{a_y\sqrt{\pi}} \frac{\Gamma(1.25)}{\Gamma(1.75)} \approx \frac{1.1128\lambda}{\sqrt{a_y}} \end{aligned} \tag{10}$$

where a_y is the amplitude of variable $\hat{y}(t)$, $a_y \le s_0$ (that is considered the most important range of the amplitude values for the analysis of the steady state), and Γ is the gamma-function defined as follows:

$$\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} \, \mathrm{d}t \tag{11}$$

Taking into account both control components, the DF of the super-twisting algorithm can be written as:

$$N = N_1 + N_2 = \frac{4\gamma}{\pi a_y} \frac{1}{j\omega} + 1.1128 \frac{\lambda}{\sqrt{a_y}} \tag{12}$$

Note that the DF of the super-twisting algorithm depends on both: the amplitude and the frequency values. The parameters of the limit cycle can be found via the solution of the complex equation.

$$W(j\Omega) \cdot N(a_y, \Omega) = -1 \tag{13}$$

where Ω is the frequency, which provides solution to the system, $W(s) = \mathbf{C}(sI - \mathbf{A})^{-1}\mathbf{L}e^{-\tau s}$. Write equation (13) in the form, $N(a_y, \Omega) = -W^{-1}(j\Omega)$, where $N(a_y, \Omega)$ is given by

$$\frac{4\gamma}{\pi a_y} \frac{1}{j\Omega} + 1.1128 \frac{\lambda}{\sqrt{a_y}} = -W^{-1}(j\Omega) \tag{14}$$

Considering the real part of both sides we can obtain:

$$\frac{1.1128\lambda}{\sqrt{a_y}} = -\mathrm{Re}\, W^{-1}(j\Omega) \tag{15}$$

Express a_y from the above equation and substitute this value in the equation, which can be obtained by considering the imaginary parts of the previous complex equation. Finally, one equation with one unknown variable Ω can be obtained as follows:

$$\Psi(\Omega) = \frac{4\gamma}{\pi\Omega} \frac{1}{\mathrm{Im}\, W^{-1}(j\Omega)} - \left(\frac{1.1128\lambda}{\mathrm{Re}\, W^{-1}(j\Omega)} \right)^2 = 0. \tag{16}$$

The amplitude a_y can be computed as follows:

$$a_y = \frac{4}{\pi\Omega} \frac{1}{\mathrm{Im}\, W^{-1}(j\Omega)}. \tag{17}$$

Therefore, if a periodic motion occurs its parameters can be found from (16) and (17). Determination of the equivalent delay from the known execution period, and therefore frequency Ω, can be done from equation (16), where $W(j\Omega)$ should be considered a function of the unknown delay.

4. Propagation of slow inputs through the observer dynamics

Chattering represents the fast component of the motion in the observer dynamics. Slow motions exist along with chattering – due to external inputs. It is known that fast oscillations have a linearizing effect on the propagation of the slow motions through the system nonlinearities. This effect is known as "chatter smoothing" phenomenon. Therefore, it is convenient to describe propagation of the slow signals through the oscillating system using the concept of the *equivalent gain* of the nonlinearity (Hsu and Meyer 1968). Essentially, the equivalent gain is the ratio of the averaged (on the period of chattering) values of the output and input of the nonlinearity. Transformation of the original dynamics of the super-twisting algorithm into equivalent dynamics for the averaged motions is illustrated below by figures 2 and 3.

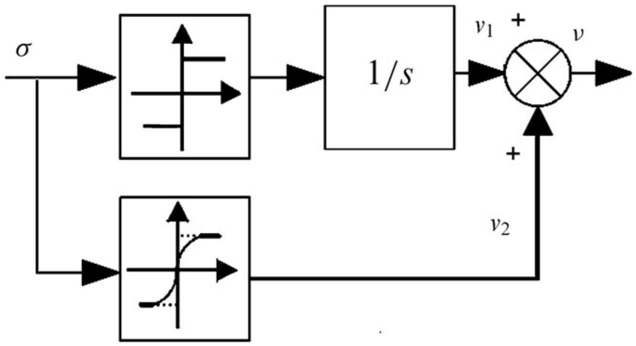

Figure 2. Block diagram of the super-twisting algorithm.

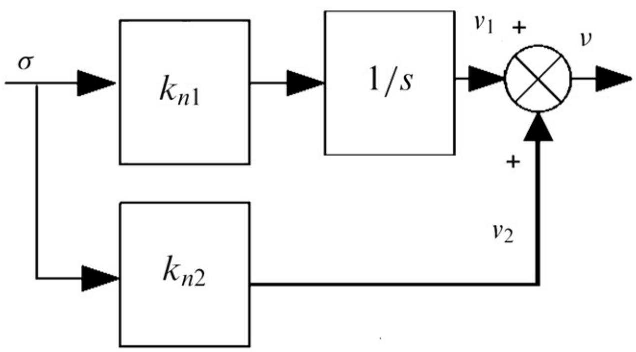

Figure 3. Linearized model of the super-twisting algorithm.

Now the objective, therefore, is to obtain the values of the equivalent gains k_{n1} and k_{n2} for the two nonlinearities of the ST algorithm.

The equivalent gain k_n for each nonlinearity can be defined as follows:

$$k_{ni} = \left.\frac{\partial \nu_{i0}}{\partial \sigma_0}\right|_{\sigma_0=0}, \quad i = 1, 2 \tag{18}$$

where ν_{10} and ν_{20} are the averaged outputs of the first and the second nonlinearities respectively, and σ_0 is the averaged input to those nonlinearities. As a result, for the first nonlinear function:

$$k_{n1} = \frac{2\gamma}{\pi a_y} \tag{19}$$

where γ is the amplitude of the relay and a_y is the amplitude of chattering at the relay input. And for the second nonlinearity, we can write:

$$\nu_{20} = \frac{\lambda}{2\pi}\int_0^{2\pi} (\sigma_0 + \alpha_y \cdot \sin\psi)^{1/2} \mathrm{d}\psi \tag{20}$$

Taking the derivative of (20) results in the following formula:

$$k_{n2} = \lambda \frac{\Gamma(1/4)}{2\sqrt{a_y}\sqrt{\pi}\Gamma(3/4)} \approx \lambda \frac{0.8346}{\sqrt{a_y}} \tag{21}$$

With both equivalent gains available we can easily build the linearized model of the observer–plant dynamical system. For that purpose, the nonlinear functions of the super-twisting controller need to be replaced with the respective equivalent gains (figure 3), and the subsequent analysis can be done as the one of a linear system. The equivalent gains are functions of the amplitude of chattering, which was determined at the analysis of chattering.

5. Examples

Consider an example of performance analysis of the SOSM observer. Let the plant be the second-order system:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}u \\ y &= \mathbf{C}\mathbf{x}, \end{aligned} \tag{22}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -1 & -3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

and the observer dynamics is given as follows:

$$\begin{aligned} \dot{\widehat{\mathbf{x}}} &= \mathbf{A}\widehat{\mathbf{x}} + \mathbf{B}u + \mathbf{L}\nu \\ \widehat{y} &= \mathbf{C}\widehat{\mathbf{x}} \end{aligned} \tag{23}$$

where

$$\mathbf{L} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

Let us consider that the equivalent delay has been found and is equal to $\tau = 0.01$ s. Determine the response of the system under observation and the observer dynamics to the harmonic oscillation of variable frequency: $ut = \sin(\omega t)$

Write an expression for the transfer function of the internal model of the observer:

$$W_l(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{L}e^{-\tau s} = \frac{5s+6}{s^2+3s+1}e^{-0.01s} \tag{24}$$

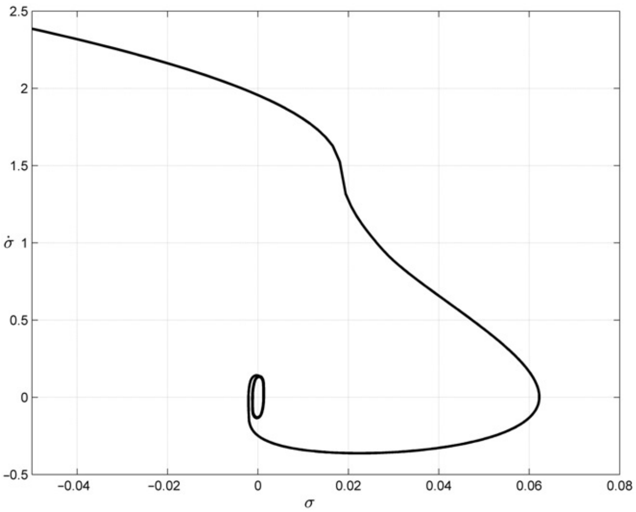

Figure 4. Super-twisting trajectory of the observer system.

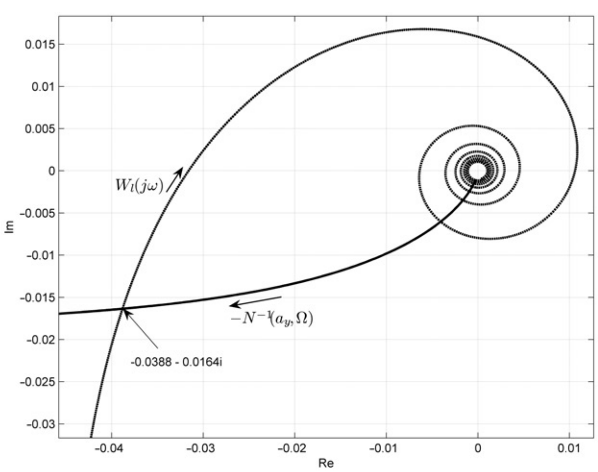

Figure 5. Point of intersection of the function $1/N(a_y)$ and of the Nyquist plot $W_l(j\omega)$ for the example.

Carry out analysis of periodic motions in the systems with the super-twisting SOSM observer if the parameters of the algorithm are given as: $\rho = 0.5$, $\gamma = 0.8$, $\lambda = 0.6$. The harmonic balance equation (13) with the transfer function given by (24) has the solution (16): $\Omega = 118.73\,\mathrm{rad\,s^{-1}}$. From (17) the amplitude of chattering can be found as $a_y = 9.30\cdot10^{-4}$. This matches well with the simulation: $\Omega_{\mathrm{sim}} = 117.24\,\mathrm{rad\,s^{-1}}$, $a_{y\mathrm{sim}} = 9.76\cdot10^{-4}$. The convergence of the observation error to a limit cycle is shown in figure 4. Finding parameters of chattering is illustrated by figure 5. The steady oscillatory modes for the plant output and super-twisting controller output are presented in figures 6 and 7 respectively.

Now let us obtain the linearized model of the observer dynamics for the averaged motions – via replacement of the two nonlinearities with two equivalent gains. Compute the gain values as per (19) and (21): $k_{n1} = 547.63$ and $k_{n2} = 16.421$.

With the nonlinear functions replaced with the equivalent gains, we can carry out analysis of the observer dynamics as its response to the harmonic signal of variable frequency. Write expression for the transfer function from input $u(t)$ to the error signal $\sigma(t)$, $W_{u-\sigma}(s)$, as follows:

$$W_{u-\sigma}(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} \times \frac{1 - e^{-\tau s}}{1 + (k_{n2} + k_{n1}\cdot 1/s)\mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{L}e^{-\tau s}} \tag{25}$$

Figures 8 and 9 give the transient response of the SOSM observer. Figure 10 gives the frequency response of the observer. The characteristics such as the bandwidth, resonant frequencies and others can be easily obtained from the transfer function (25) of the linearized plant–observer model.

The observation error can also be computed for x_1 and x_2 using the transfer functions from the system input to the difference of the variable and its estimate as (26) for x_1 and (27) for x_2. The amplitude–frequency characteristics corresponding to (26) and (27) are presented in figure 11.

$$W_{u-\Delta x1}(s) = \frac{\mathbf{C}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}(1-e^{-\tau s})(k_{n2}+k_{n1}/s)\mathbf{C}_1(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{L}e^{-\tau s}}{1+(k_{n2}+k_{n1}/s)\mathbf{C}(s\mathbf{I}-\mathbf{A}])^{-1}\mathbf{L}\mathbf{e}^{-\tau \mathbf{s}}} \tag{26}$$

where $\mathbf{C}_1 = [1 \quad 0]$

$$W_{u-\Delta x2}(s) = \frac{\mathbf{C}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}(1-e^{-\tau s})(k_{n2}+k_{n1}/s)\mathbf{C}_2(s\mathbf{I}-\mathbf{L})^{-1}\mathrm{L}e^{-\tau s}}{1+(k_{n2}+k_{n1}/s)\mathbf{C}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{L}e^{-\tau s}} \tag{27}$$

where $\mathbf{C}_2 = [0 \quad 1]$

6. Conclusion

The problem of obtaining frequency-domain characteristics of dynamical accuracy of sliding mode observers that utilize the SOSM control algorithms is presented in the article. The problem is solved with the use of the describing function method and of the

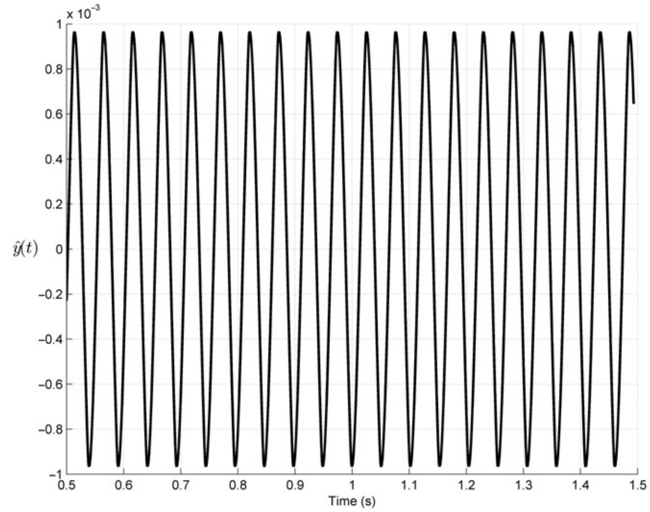

Figure 6. Observer output $\hat{y}(t)$ for $u(t)=0$.

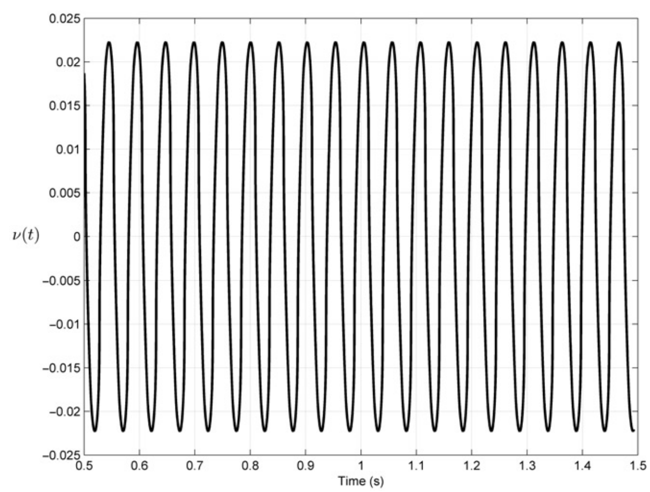

Figure 7. Output v of super-twisting algorithm for $u(t)=0$.

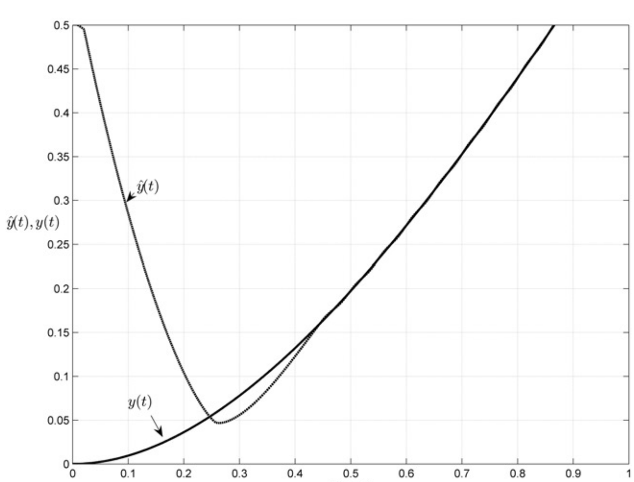

Figure 8. Output of the plant $y(t)$ in (lower plot) and output tracking for observer.

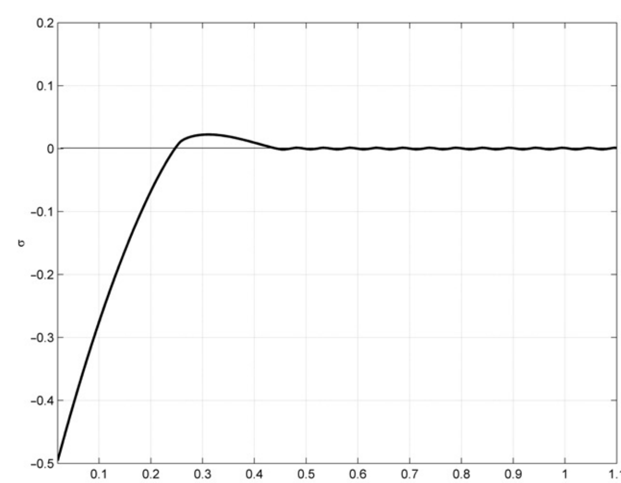

Figure 9. Observation error $\sigma = y - \widehat{y}$ (the initial condition $y=0.5$ and $u(t)=\sin(t)$.

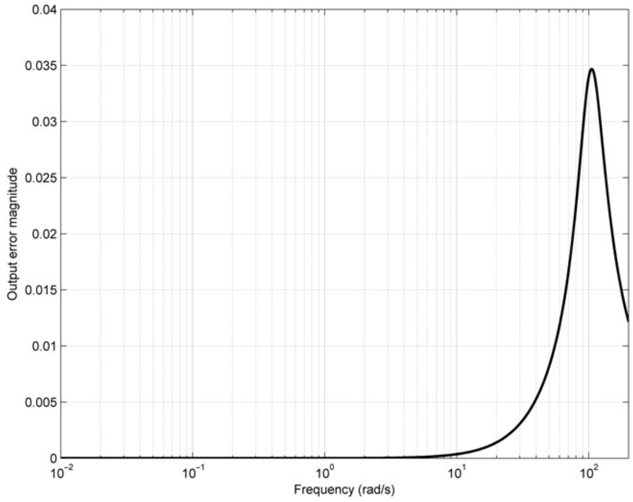

Figure 10. Error magnitude *vs*. frequency.

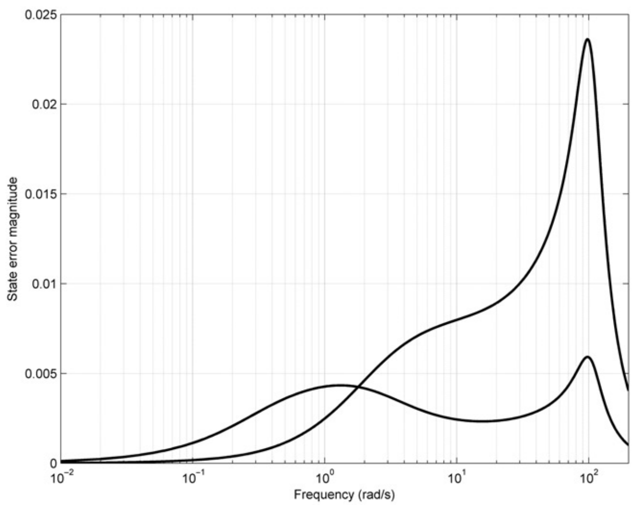

Figure 11. Observation error for x_1 (lower plot) and for x_2 (upper plot) *vs*. frequency.

concept of the equivalent gain of the nonlinearity. This concept is extended to the SOSM algorithms. The problem is solved with the use of a two-step procedure.

At the first step, the problem of finding the parameters of chattering is solved, and after that at the second step the input–output problem with respect to the oscillatory observer dynamics is solved with the use of the equivalent gain concept. As a result, such characteristics of the observer as the transfer function and the frequency response for the observation errors can be obtained. This provides a straightforward methodology of the SOSM observer performance evaluation and design.

Acknowledgements

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