Output tracking of systems subjected to perturbations and a class of actuator faults based on HOSM observation and identification

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Abstract

This paper deals with the output tracking problem of a MIMO system subjected to a class of actuator faults and unmatched perturbations. The proposed methodology is based on high order sliding mode observation and identification techniques. A dynamic sliding surface is proposed using a backstepping-like design strategy in order to counteract the effects of the unmatched perturbations. Whereas a continuous sliding mode control is designed to steer the states toward the sliding surface. The identified value of the fault is injected to alleviate the control gain while accomplishing fault accommodation. As a consequence, the chattering is attenuated. A simulation example for a 3-DOF helicopter highlights the efficiency of the present method.

Key words: Disturbance rejection, fault-tolerant systems, sliding-mode control

1 Introduction

Many techniques have been developed for fault detection, isolation and compensation to achieve fault-tolerance in controlled systems, see for example [1–3]. In this context, sliding mode control (SMC) has efficient characteristics thanks to its insensibility to matched perturbations, i.e. those which appear implicitly at the input channels [4]. This property has been taken into account to withstand with actuator faults in a wide class of applications. [5–7]. In [5], fault detection and isolation is achieved by means of a residual generation scheme and thresholds definition. The fault accommodation takes place using the healthy redundant actuators. The robustness of the method may be spoiled if a fault occurrence is disregarded due to an inappropriate threshold selection. This is not the case of [6,7], where the fault information appears as a given weighting matrix which contains the effectiveness level of the actuators. Thus a control allocation is proposed to accommodate the control among the actuators.

Such strategies achieve the proposed goal assuming full state information, whereas in [7] the results are extended considering only output information and using an integral sliding mode control scheme. Still, the aforementioned approaches are tarnished by the so-called chattering effect. To overcome this issue, High Order Sliding Mode (HOSM) techniques have been considered. In [8] a continuous fault tolerant control allocation is proposed based on HOSM observers. On the other hand, strategies based on fault reconstruction and compensation using HOSM schemes are developed in [9–11]. In [9] the actuator fault is reconstructed using a HOSM differentiator and then the identified signal is injected in the control law in order to compensate its effects. Nevertheless, it cannot stand for disturbances affecting the sub-actuated dynamics.

Strategies explicitly designed to cope with unmatched perturbations, i.e. those which are not implicit in the input channels, have been proposed in [12,10,11,13]. In [13] the unmatched perturbations and faults are estimated by means of an adaptive mechanism whereas a backstepping technique is applied to compensate its effects. The performance of the closed-loop system is affected by the convergence rate of the adaptation mechanism. HOSM identification based strategies circumvent this issue owing to its finite time convergence capabilities. In this direction, in [10] a smooth backstepping control based on exact identification is proposed. This technique consider full state measurements and exploits...
a HOSM differentiator to identify the perturbations at each coordinate. Moreover, additional HOSM differentiators are involved to facilitate the control law computation. In [12,11] output regulation strategies are introduced. While a priori knowledge of the perturbation dynamics is assumed in [12], in [11] perturbations are tackled as unknown inputs. Thus, a dynamic sliding surface is designed to cope with the unmatched perturbations. While a discontinuous control counteracts the matched perturbations. As a consequence, chattering may occurs.

The goal of the paper is to design an observed-based control for a MIMO system affected by unmatched perturbations and subjected to a class of actuator faults. The effect of the perturbations and actuator faults are thus handled as additive time-varying unknown inputs that can be identified by a HOSM observer [14,15]. To achieve the control design, first a finite time convergent HOSM observer is used to estimate the state and identify the actuator faults as well as the unmatched perturbations and their successive derivatives based on the available output measurements [16]. Then, a dynamic sliding surface is designed to accomplish the desired control goal (tracking) despite the unmatched perturbations using a backstepping approach. Finally, a super-twisting based control law steers the state trajectories to the sliding surface guaranteeing that there remain, the identified values of the faults are injected to alleviate the controller gain accomplishing fault accommodation.

The contribution of this paper is threefold: i) the output tracking problem of a MIMO system affected with actuator faults and unmatched perturbations is achieved. In this way ii) the backstepping compensation design is accomplished for the sliding surface design, i.e. for the reduced order system, diminishing the number of the derivatives involved and with this the computation complexity; iii) finally, it is shown that the proposed solution possesses fault accommodation capability against additive time-varying actuator faults.

The paper is organized as follows. In Section 2, some preliminaries are given; the state estimation and perturbation identification by a HOSM observer are summarized. In Section 3 the problem formulation and control challenge are presented. Section 4 presents the control strategy. The performance of the proposed technique is illustrated through numerical simulations for a 3-DOF Helicopter in Section 5. Finally some concluding remarks are given in Section 6.

2 Preliminaries

The following notation is used. For a matrix \( X \in \mathbb{R}^{n \times m} \) with \( \text{rank}(X) = r \), the matrix \( X^+ \in \mathbb{R}^{n \times r} \) with \( \text{rank}(X^+) = n - r \) is defined such that \( X^+X = 0 \). If \( X \in \mathbb{R}^{n \times m} \) with \( \text{rank}(X) = m \), the matrix \( X^+ = (X^T X)^{-1}X^T \) is defined as the pseudo-inverse of \( X \). For any \( X \in \mathbb{R}^n \) the symbol \( |x| \) denotes its absolute value. If \( x \in \mathbb{R}^n \) then \( |x| \) states for the Euclidean norm. If \( X \in \mathbb{R}^{n \times m} \) is a matrix, the symbol \( |X| \) denotes the corresponding induced norm.

Consider the particular class of faulty systems that can be modeled as a linear time invariant system affected by unknown inputs

\[
\begin{align*}
\dot{x} &= Ax + B(u + f) + Gv \\
y &= Cx
\end{align*}
\]

where \( x \in \mathbb{R}^n \), \( y \in \mathbb{R}^p \) (\( 1 \leq p < n \)) represent the system’s states and measured outputs respectively. The vector \( u \in \mathbb{R}^m \) represents the input and \( v \in \mathbb{R}^q \) \( (q \leq n - m) \) are the perturbations. The matrices \( A, B, G \) and \( C \) are of the corresponding dimensions. \( f \in \mathbb{R}^m \) refers to the (actuator) faults and behaves as the particular class of additive faults, see [17] for more details on modeling faults. The following is assumed henceforth: A1) the system is strongly observable, or equivalently the triplet \( (A, C, [B \ G]) \) has no zeros; A2) \( \text{span}(G) \subset \text{span}(B^\perp) \); A3) \( v \) and its derivatives up to order \( r \) are bounded, i.e. \( |v| \leq \nu_0 \) as well as \( |v^{(i)}| \leq \nu_i \) for \( i = 1,\ldots, r \), for all \( t \geq 0 \); A4) \( f \) and its derivatives up to order \( r - 2 \) are bounded, i.e. \( |f| \leq \nu_0 \) as well as \( |f^{(i)}| \leq \nu_i \) for \( i = 1,\ldots, r - 2 \), for all \( t \geq 0 \).

Notice that A2 states that faults and perturbations belong to different subspaces. Hence, in A3 the faults smoothness assumption is relaxed w.r.t. perturbations signals in A4.

Following [16], a HOSM observer is introduced to reconstruct the state vector and identify the actuator faults, the perturbations as well as the perturbations’ successive derivatives.

2.1 High Order Sliding Mode Observer

First, a dynamic auxiliary system is proposed to bound the observation error, i.e. \( \dot{\bar{x}} = A\bar{x} + Bu + L(y - C\bar{x}), \bar{x} \in \mathbb{R}^n \), the gain \( L \) is designed such that \( \bar{A} := A - LC \) be Hurwitz. Let \( e = x - \bar{x} \) whose dynamics follows

\[
\dot{e} = \bar{A}e + Dw
\]

where \( D = [B \ G] \) and \( w = [f^T \ \nu^T]^T \) and \( \nu_c = Ce \). Thus, under the assumptions A3 and A4, the unknown inputs vector \( w \) and its successive derivatives, are also bounded. Thus, it is well known that \( e \) will have a bounded norm, i.e. \( |e| < \bar{e} \) for all \( t > T_c \).

The error vector will be represented as an algebraic expression of the output and its derivatives. To this aim, a decoupling algorithm is involved in order to get rid of the effects of the unknown input vector \( w \).

Starting with \( M_1 := C \) and \( J_1 := (M_1D)^\perp \), let \( M_\kappa \) be defined in a recursive way in the following form,

\[
M_\kappa = \begin{bmatrix}
(M_{\kappa-1}D)^\perp M_{\kappa-1}\bar{A} & M_{\kappa-1}D \\
M_1 & \end{bmatrix} \quad J_{\kappa-1} = (M_{\kappa-1}D)^\perp \begin{bmatrix}
J_{\kappa-2} & 0 \\
0 & I_p
\end{bmatrix}
\]
Due to A1, there exists a unique positive integer $\kappa \leq n$ such that the matrix $M_k$, generated recursively, satisfies the condition $\text{rank}(M_k) = n$ (see [16]). Therefore, the following algebraic expression can be constructed

$$M_k e = \frac{d^{k-1}}{d t^{k-1}} \left[ \begin{array}{c} y_k \\ \vdots \\ y_{k-1} \\ y_e \end{array} \right] = \frac{d^{k-1}}{d t^{k-1}} (4)$$

where $y_i^j$ represents the $i$-th anti-derivative of $y$, that is, $y_i^j = \int_0^t \cdots \int_0^{t_i} y_e \, d \tau_i \ldots dt$. From the above equation a solution for $e$ exists, i.e. $e = M_k^{-1} \frac{d^{k-1}}{d t^{k-1}} Y$.

Thus, a real-time HOSM differentiator can be used to provide exact and finite time differentiation of $Y$ [18]. It is given by

$$\dot{z}_{i,0} = \lambda_0 Y^1 |z_{i,0} - Y|^\tau \text{sign}(z_{i,0} - Y) + z_{i,1}$$

$$\dot{z}_{i,j} = \lambda_j Y^1 |z_{i,j} - z_{i,j-1}|^\tau \text{sign}(z_{i,j} - z_{i,j-1}) + z_{i,j+1}$$

$$\dot{z}_{i,\ell} = \lambda_\ell Y^1 |z_{i,\ell} - z_{i,\ell-1}|^\tau \text{sign}(z_{i,\ell} - z_{i,\ell-1})$$

with $j = \ell, \ell-1$. $\ell$ is the differentiator order. The differentiator input $Y_i$ for $i = \ell, n$ represents the components of $Y$. A positive sequence of $\lambda_j$ can be selected as in [18]. The gain $Y$ is a Lipschitz constant of $Y(\bar{t})$. Due to assumption A3, the higher differentiator order may be $\ell = \kappa + r - 3$, i.e. $|e^{(\ell-1)}| < Y$. Therefore, $Y \geq |\bar{A}|^{r-1} |\bar{B}| + |\bar{G}| |\bar{Y})|$. In [18], it was shown that with the proper choice of the constants $\lambda_j$ and $Y$ for all $j = 0, \ell$ there is a finite time $t > t_0$ such that $z_j = \frac{d \ell}{d \ell} Y$ is fulfilled for all $j = 0, \ell$, $z_j = [z_{i,1}, \ldots, z_{i,n}]^T$. Hence, the vector $e$ in (4) is recovered from the $(\kappa - 1) - \ell$th sliding dynamics, i.e. $e = M_k^{-1} \dot{z}_{k-1}$ holds for $t > t_k$. Consequently the next expression holds

$$\dot{x} := M_k^{-1} \dot{z}_{k-1} + \dot{x}$$

where $\dot{x} \in \mathbb{R}^n$ is the estimated value of $x$ for all $t > t_k$.

2.2 Faults and perturbations identification

The perturbations and actuator faults vectors $v$ and $f$ can be recovered from (3), where $e$ can be obtained from the HOSM differentiator (5), i.e. the equality $z_k = \dot{x}$ is accomplished for all $t > t_k$. Hence, working out (5) it yields to

$$\dot{f} = (G^1 B)^+ G^1 (z_k - \dot{A} z_{k-1})$$

where $\dot{f} \in \mathbb{R}^m$ represents the identified value of $f$. Whereas for the perturbations and its successive derivatives it yields to

$$\dot{\phi}^j = (B^1 G)^+ B^1 (z_{k+j} - \dot{A} z_{k+j-1})$$

for $j = 0, r - 3$, where $z_{k+j}$ comes from (5). Therefore, $\dot{\phi}^j \in \mathbb{R}^4$ represents the estimate of $\phi^j$ for all $t > t_o$.

The next derivative $\dot{\psi}^{(r-2)}$ may be achieved from $\dot{\psi}^{(r-2)} = (B^2 G)^+ B^1 (e^{(r-1)} - \dot{A} z_{k+r-2})$ where $B^2 e^{(r-1)}$ can be recovered by means of $B^2 e^{(r-1)} = B^2 \frac{d}{dt} z_{k+r-2}$. In order to do that, the Lipschitz constant may be calculated from $|B^2 e^{(r-1)}| < \Upsilon_2$, i.e. $\Upsilon_2 \geq |B^2 G| + |B^2 G| \dot{\Upsilon}_{r-1}$. Thus, with the proper choice of $\lambda_j$, it follows from [18] $\dot{\psi}^{(r-2)} = \psi^{(r-2)}$ for all $t > t_o > t_4$.

3 Problem statement

Consider a strict feedback form representation of system (1), see [19,20]. Due to the pair $(A, B)$ is controllable, system (1) can be always reduced to the block-controllable form, it is composed into $r$ connected sub-systems, i.e.

$$\dot{x}_1 = A_1 x_1 + B_1 (x_2 + \Gamma_1 \nu)$$

$$\dot{x}_2 = A_2 x_2 + B_2 (x_3 + \Gamma_2 \nu)$$

$$\dot{x}_r = A_r x_r + B_r (u + f)$$

for $i = 2, r - 1$, where $\nu = [x_1^T \cdots x_r^T]^T$, $x_i \in \mathbb{R}^{m_i}$, $n_i = \text{rank}(B_i)$, $\sum_{i=1}^r n_i = n$. The state vector $x \in \mathbb{R}^n$ is given by $\dot{x} = [x_1^T \cdots x_r^T]^T$. The output signal is given by (2) while the controlled output is given by $x_1 \in \mathbb{R}^{m_1}$. The sub-system (9)-(10) for $i = 1, r - 1$ represents the sub-actuated dynamics and (11) corresponds to the actuated dynamics thus, $x_r \in \mathbb{R}^m$. The matrices $A_i, \Gamma_i$ are of the corresponding dimensions. Further details can be found in [21].

Control aim. The goal is to design an output feedback continuous sliding mode controller $u$ which allows the coordinate $x_r$ to track a smooth signal $x_{d_r}$ in spite of the perturbations $v$ and the occurrence of the faults $f$.

The sliding control design relies on the availability, in finite time, of the exact estimation of the state and the identification of the faults as well as perturbations and their successive derivatives until $r - 2$-th order. Thus, from (6), after $t > t_o$, the identities $\dot{x}_1 = x_1, \ldots, \dot{x}_r = x_r$ are certainly obtained.

4 Control Design

First, the design of the sliding surface is achieved in $r - 1$ steps, it is carried such that the sub-actuated dynamics (9)-(10) accomplishes the desired behavior in spite of the perturbations. Then, at the $r-th$ step, the control law is conceived such that the state trajectories reach the sliding surface and there remain. It is assumed that the successive derivatives of the desired trajectory $x_{d_r}$ are available.

Step 1. Starting with sub-system (9), the coordinate $x_2$ can be exploited as a fictitious control input $x_2 := \phi_1$,

$$\dot{\phi}_1 = -\Gamma_1 \dot{\nu} - B_1^1 (A_1 x_1 - \dot{\bar{A}}_1 (x_1 - x_d) - \dot{x}_d)$$

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where \( \hat{A}_1 \in \mathbb{R}^{n_1} \) is a Hurwitz matrix containing the desired convergence performance of \( x_1 \) towards \( x_d \). Notice that since \( \text{rank}(B_i) = n_i \) then \( B_i B_i^T \) is invertible, thus \( B_i^T \) is the right-inverse matrix of \( B_i \).

**Step i.** The coordinate \( x_{i+1} \) is a fictitious control input for (10), it is \( x_{i+1} := \phi_i \).

\[
\phi_i = -\Gamma \hat{\phi} - B_i^T (A_i \hat{x}_i - \hat{A}_i (x_i - \phi_{i-1}) + X_{i-1} (x_{i-1} - \phi_{i-2}) - \phi_{i-1})
\]

(13)

for all \( i = \sum T - 1 \), \( \hat{A}_i \in \mathbb{R}^{n_i} \) is a Hurwitz matrix and \( X_{i-1} = P_i^{-1} B_i^T P_{i-1} \), with \( P_i \) a positive definite matrix satisfying \( P_i \hat{A}_i + \hat{A}_i^T P_i = -I \).

**Step r.** Finally, the sliding surface \( s \) is designed like

\[
s = x_r - \phi_{r-1}
\]

(14)

with \( s \in \mathbb{R}^{n_m} \) and the control law becomes

\[
u = -B_r^T \left( A_r \hat{x}_r - \hat{A}_r \phi_r - B_{r-1}(x_{r-1} - \phi_{r-2}) + u \right) - \hat{f}
\]

(15)

where \( \hat{\phi} \) is a linear combination of the state \( x \) and \( \hat{\nu}, \ldots, \hat{\nu^{(r-2)}} \). The control \( u \in \mathbb{R}^m \) is the super-twisting control (see (22)) given by

\[
u = K_1 \Psi_1 + K_2 \int_0^t \Psi_2 dt
\]

(16)

with the matrices \( K_j = \text{diag} \left( k_{j_1}, \ldots, k_{j_m} \right) \) for \( j = \sum 2 \), \( \Psi_1 = \text{diag} \left( \Psi_{11}, \ldots, \Psi_{1m} \right) \) and \( \Psi_2 = \text{diag} \left( \Psi_{21}, \ldots, \Psi_{2m} \right) \), where \( \Psi_{1j} = |s_j| \frac{1}{2} \text{sign}(s_j) \) and \( \Psi_{2j} = \text{sign}(s_j) \) for \( j = \sum m \).

Following [23], the components of the matrices gains \( K_1 \) and \( K_2 \) are given as

\[
k_{1j} > 0 \quad k_{2j} > 3 \phi + 2 \left( \frac{\phi}{\kappa_j} \right)^2
\]

(17)

for \( j = \sum m \), the constant \( |B_{r-1}^T \cdots B_1^T \Gamma_1^{r-1}| \leq \phi \).

Now, the stability of the closed loop system is achieved in two stages. First, the convergence of the trajectories to the sliding surface \( s = 0 \) is shown. Then, a composite Lyapunov function is designed to demonstrate the convergence of the sub-actuated dynamics.

For the sake of simplicity, the convergence of the individual components of the sliding surface is studied. Differentiating (14) and applying the control law (15) it yields to

\[
s_j = -k_{j1} \Psi_{1j} - k_{j2} \int_0^t \Psi_{2j} dt - \omega_j
\]

with \( j = \sum m \); \( \omega_j \), \( s_j \), represent the components of vectors \( s, \omega \). The vector \( \omega = -B_{r-1}^T \cdots B_1^T \Gamma_1^{r-1} \) is an uncertainty due to the effects of the unmatched perturbation in the actuated dynamics.

Defining \( \mu_j = -k_{j2} \int_0^t \Psi_{2j} dt + \omega_j \), yields to

\[
s_j = -k_{j1} \Psi_{1j} + \mu_j \quad \mu_j = -k_{j2} \Psi_{2j} + \omega_j
\]

(18)

where \( \omega = B_{r-1}^T \cdots B_1^T \Gamma_1^{r} \) which under \( A_3 \) is bounded, i.e. there exists a constant \( \phi \) such that \( |\omega| \leq \phi \). In [23], it was stated that the auxiliary system (18) can be stabilized by selecting the controller gains as in (17) achieving \( s = 0 \) in finite time. As a consequence, from (14), it follows that \( x_r = \phi_{r-1} \).

Now, the sub-actuated dynamics is examined for the second part of the convergence exposition. Notice that \( x_{i+1} = \phi_i \) is exploited as a fictitious control for the coordinate \( x_i \). The error variables \( \sigma_1 = x_1 - x_d, \sigma_i = x_i - \phi_{i-1} \) for \( i = 2, \sum T - 1 \) are defined. A composite Lyapunov function for the auxiliary error dynamics is constructed henceforth.

First, consider \( x_2 \) as a pseudo-control to achieve \( x_1 \rightarrow x_d \). Hence, a Lyapunov function candidate given by \( V_1 = \sigma_1^T P_1 \sigma_1 \) is proposed. Differentiating it along the time yields to

\[
\dot{V}_1 = 2 \sigma_1^T \left( A_1 \sigma_1 + B_1 (x_2 + \Gamma_1 v) - \sigma_1 \right)
\]

Since \( x_2 = \phi_1 + \sigma_2 \) and taking into account (12), it yields to

\[
\dot{V}_1 = -2 |\sigma_1|^2 + 2 \sigma_1 P_1 B_1 \sigma_2
\]

From (9)-(10) the following auxiliary system occurs

\[
\sigma_1 = \hat{A}_1 \sigma_1 + B_1 \sigma_2
\]

\[
\sigma_2 = A_2 \hat{x}_2 + B_2 (x_3 + \Gamma_2 v) - \phi_1
\]

(19)

(20)

where \( x_3 \) is seen as a pseudo-control to stabilize (19)-(20) at zero. Considering the Lyapunov candidate \( V_2 = \sigma_2^T P_2 \sigma_2 \), its derivative with respect to time returns

\[
\dot{V}_2 = V_1 + \sigma_2^T P_2 \left( A_2 \sigma_2 + B_2 (x_3 + \Gamma_2 v) - \phi_1 \right)
\]

Given that \( x_3 = \phi_2 + \sigma_3 \) and in virtue of (13), it yields to

\[
\dot{V}_2 \leq -|\sigma_2|^2 + 2 \sigma_2 P_2 B_2 \sigma_2
\]

This procedure is followed for each sub-actuated coordinate. From (9)-(10) results, at \( r - 1 \) step, the sub-actuated error dynamics

\[
\sigma_1 = \hat{A}_r \sigma_1 + B_1 \sigma_2
\]

\[
\sigma_2 = A_r \hat{x}_r + B_r (x_{r+1} + \Gamma_r v) - \phi_{r-2}
\]

(21)

(22)

(23)

As a consequence, a composite Lyapunov function candidate is given by \( V_\sigma = \sum_1^r \sigma_j^T P_j \sigma_j \). Its derivative with respect to time produces \( \dot{V}_\sigma = -\sum_1^r |\sigma_j|^2 + 2 \sum_1^r \sigma_j^T P_j B_r \sigma_{r-2} + 2 \Gamma_r^T (A_r \sigma_{r-1} + B_r (x_r + \Gamma_r v) - \phi_{r-3}) \). Due to \( s \equiv 0 \), therefore \( x_r \equiv \phi_{r-1} \) and in virtue of (13), the derivative of \( V_\sigma \) yields to

\[
\dot{V}_\sigma = -\sum_1^r |\sigma_j|^2
\]

As a result, \( \sigma_i \) for \( i = r - 1, T \) converges asymptotically to zero. Consequently, \( x_i \rightarrow \phi_{i+1} \) for \( i = r - 1, T \). Therefore, \( x_1 \rightarrow x_d \).
5 Example

Consider a 3-DOF helicopter, see [24]. The system is composed by two arms: a small arm with a propeller at each end connected with an arm swinging in a fixed base (see Fig. 1). The system can rotate freely around three axes. A linearized model around $\epsilon^*_3 = 0$ is given by

$$\dot{\epsilon}_1 = 0.45((u_1 + f_1) + (u_2 + f_2)) \quad (24)$$

$$\dot{\epsilon}_2 = 3.05((u_1 + f_1) - (u_2 + f_2)) \quad \epsilon_3 = -0.49\epsilon_2 + v$$

where $\epsilon_1, \epsilon_2, \epsilon_3 \in \mathbb{R}$ represent respectively the elevation, pitch and travel angles and $u \in \mathbb{R}^2$. The system is subjected to the actuator faults $f_1, f_2$. A disturbance signal $v$ is acting on the travel dynamics. The measured outputs correspond to the angular positions, i.e. $y = [\epsilon_1 \, \epsilon_2 \, \epsilon_3]^T.$

The goal is that the travel coordinate $\epsilon_3$ tracks a desired trajectory $x_d = 5\sin(1.1t)$ in spite of the faults and disturbances affecting the system. The simulation trial consists in four stages: (a) for $0 \leq t < 60$ the system is working free of faults and disturbances (i.e. $f = [0 \, 0]$, $v = 0$); (b) during $60 \leq t \leq 140$ a disturbance appears $v = 2\sin 3t + 1.5$; (c) in $100 \leq t$ a type of liquid oscillatory fault occurs in the first actuator $\dot{f}_1 = \cos(1.5t) + 5$; (d) for $140 \leq t$, a drifting fault (i.e. $\dot{f}_2 = \beta t : \beta > 0$) appears in the second actuator. The simulation sampling time is $100[\mu s].$

Notice that $f_1$ and $f_2$ represent realistic faults that may affect actuator servo-loops in flight systems, see [17]. According to aeronautics terminology, liquid failures correspond to additive oscillatory faults.

Observer design. Firstly, the state vector is selected as $x = [\epsilon_1 \, \epsilon_2 \, \epsilon_3 \, \dot{\epsilon}_1 \, \dot{\epsilon}_2 \, \dot{\epsilon}_3]^T.$ Then, it can be easily shown that for the obtained state space representation, the triplet ($A, C, D$) is strongly observable. Thus, regarding (3), an auxiliary system matrix $\hat{A}$ with eigenvalues $\{1.6, 1.5, 1.8, 1.9, 1.7, 1.1\}$ is considered. Subsequently, for $\kappa = 2$ iterations a full column rank matrix $M_2 = [-21 \, 0 \, 0 \, 1 \, 0 \, 0 \, -13 \, 0 \, 0 \, 1 \, 0 \, c]$ is found.

The differentiator in (5) has an order $\ell = 3$, such that $\dot{\hat{e}}$, $\ddot{\hat{e}}$, $\hat{v}$ and $\hat{\nu}^{(1)}$ can be estimated. The bounds are $\nu = 3.5$, $\nu_1 = 6$, $\nu_2 = 18$, $\nu_1 = 10$, $\nu_2 = 7.5$. Then, $\gamma = 150$, $\lambda_i = \{1.1, 1.5, 3.5\}$.

Control design. The controller aim is that $\epsilon_3 \to x_d$. Moreover, the elevation angle is kept at $\epsilon_x = 15[\text{deg}]$ so the variables $\dot{\hat{e}}_1 := \epsilon_1 - \epsilon_x$ and $\epsilon_3 = \int_0^t \dot{\hat{e}}_1 dt$ are introduced. As a consequence, an extended state vector is considered $x = [\dot{\hat{e}}_1 \, \epsilon_2 \, \dot{\hat{e}}_2 \, \dot{\hat{e}}_3 \, \epsilon_4]^T$. This extended system can be transformed into (9)-(11). Following the coordinates transformation given in [21] with $x_1 = \epsilon_3$, $x_2 = [\epsilon_1 \, \epsilon_4]^T$, $x_3 = [0.98 \, 0.14; \, -0.14 \, -0.98]$ $\dot{\hat{e}}_1 \, \epsilon_2]^T$, $x_4 = [\dot{\hat{e}}_1 - \dot{\hat{e}}_2]^T$ it yields to

$$\dot{x}_1 = B_1x_2$$

$$\dot{x}_2 = B_2(x_3 + \Gamma_2 v)$$

$$\dot{x}_3 = B_3x_4$$

$$\dot{x}_4 = B_4(u + f) \quad (25)$$

where $B_1 = [1 \, 0]$, $B_2 = [0.49 \, 0.07; \, -0.14 \, 0.98]$, $\Gamma_2 = [1.99 \, -0.29]^T$, $B_3 = [0.14 \, 0.98; \, -0.98 \, 0.14]$, $B_4 = [-0.45 \, -0.45; -3.05 \, 3.05]$.

The fictitious controls $\phi_i$ for $i = 1, 2$ are computed following (12)-(13). The sliding surface is thus designed as $\sigma = x_4 - \phi_3$. The control signal (15) has $\hat{A}_1 = -1.8, \hat{A}_2 = diag(-2, -3)$, $\hat{A}_3 = diag(-4, -5)$, $\hat{B} = 160$. The super-twisting gains (17) are $k_{ii} = 10$, $k_{ii} = 600$.

For comparison purposes, an $H_\infty$ controller is considered. The design of this controller is formulated as an $H_\infty$ mixed-sensitivity problem (see [25] for additional details). This framework is employed for the easy management of control design trade-offs. Weighting functions are used to achieve small damping ratio on $\epsilon_3$ with a tracking error lower than 0.01. Moreover, an additional weighting function has been used to attenuate the energy of the control signal applied to actuators such that the control signal behavior keeps smooth (i.e. it provides a high-frequency filter action).

Fig. 2 shows the tracking performance with the four trial steps highlighted. It can be seen that both controllers achieve the desired travel goal during the nominal stage, see (a). However, when disturbances and/or faults act on the system, refer to the stages (b)-(d), the compensation based controller in Fig. 2.A is capable to attain the prescribed goal with a better performance than the $H_\infty$ controller in Fig. 2.B.

The Fig. 3 shows the control signals for the first (A) and second (B) input channels, $u_1$ and $u_2$ respectively. The compensation based controller proposed in this work is displayed in pink while the black lines represent the $H_\infty$ control signals.

6 Conclusions

An output tracking scheme for a MIMO system subjected to unmatched perturbations and actuator faults was propo-
Fig. 2. Travel tracking performance: (A) proposed control and (B) $H_{\infty}$ control.

Fig. 3. Control signals comparison for both control laws: $H_{\infty}$ (solid) and compensation based control (dotted). In (A) $u_1$ is displayed while $u_2$ appears in (B).

References


