This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

Time-Varying Gain Differentiator: A Mobile Hydraulic System Case Study

Carlos Vázquez, *Member, IEEE*, Stanislav Aranovskiy, *Member, IEEE*, Leonid B. Freidovich, *Senior Member, IEEE*, and Leonid M. Fridman, *Member, IEEE*

***Abstract*— In mobile hydraulic systems, velocities are typically not measured. However, using their reliable estimates for feedback is known to allow designing better control laws. We are going to present a specialized technique to compute such estimates using measurements of positions and pressures in the chambers of hydraulic cylinders. With a rough estimate for an upper bound of the second derivative, computed online from pressures, the goal is to find the first derivative of the position signal in the presence of noise. We propose a differentiator with a continuous time-varying gain, constructed from pressure measurements, achieving chattering attenuation without compromising the performance of estimation. The gain is constructively tuned using analysis based on a time-varying Lyapunov function. In addition, the obtained ultimate bounds on differentiation errors provide a criterion for the enhancement of the precision of the proposed algorithm with a constructive design of its parameters. The experimental results over a forestry-standard mobile hydraulic crane confirm the advantages of the methodology.**

***Index Terms*— Time-Varying gain differentiator, mobile hydraulic system, second order sliding mode, high-gain oberver, on-line differentiator, velocity observer.**

I. Introduction

THE online first-order differentiation is an old problem, which has become a focus of intensive research in the recent years. High-gain observers [1] and high-order sliding mode differentiators have shown a very good performance even in the presence of noise [2]–[5]. Basically, the key breakthrough appeared in [6], where a robust first-order exact differentiator using a second-order sliding mode technique, known as supertwisting algorithm (STA), was introduced. Based on the STA, an observer for mechanical systems was presented in [7]. In order to estimate the convergence time, as in [8] and [9], a non-smooth Lyapunov-function-based approach was proposed. An exponential second-order sliding mode differentiator restricted to signals with bounded third derivative was proposed in [10]. A differentiator based on a variant of the STA equipped with a hybrid adaptation was presented in [11], ensuring global differentiation ability. Recently, some interesting remarks about the convergence time and disturbance rejection were presented in [12]. In order to attenuate the characteristic chattering phenomenon, some adaptive schemes had been developed in [13] and [14]. In [15], a variable-gain approach was proposed, achieving chattering attenuation. In [4], a second-order sliding mode algorithm that included an adaptive growing gain was presented. Uniformly convergent algorithms were designed in [16] and [17]. An adaptive STA for actuator oscillatory failure case reconstruction was proposed in [18]. Regarding hydraulic actuators, recently in [19], an algorithm with a growing gain has been introduced, with the drawback that the gain needs resetting and the acceleration case cannot be covered. Vázquez *et al.* [20] presented a design of a time-varying gain based on pressure measurements that cover the whole practical acceleration profile; however, the effect of noise was neglected in the Lyapunov analysis. Besides, to the best of our knowledge, an enhancement of differentiator parameters or any justifiable tuning rules for them under the presence of noise has not been presented for the case of a time-varying gain.

In addition, one of the main difficulties is the selection of the gain. If a global constant bound is chosen for the whole practical operation region, the constant would be excessively large that would result in increasing errors of the differentiator. Some ideas on how to include a time-varying gain in the design have been given in [21], assuming that the $(n + 1)$th-order derivative has a variable upper bound available in real time; however, no suggestions on choosing such bounds for control systems or the enhancement/tuning of the differentiator parameters under the presence of noise have being given.

Concerning the case study, mobile hydraulic systems are the main components of heavy-duty machines widely used in

Manuscript received November 26, 2014; revised June 19, 2015 and November 1, 2015; accepted December 18, 2015. Manuscript received in final form December 19, 2015. The work of C. Vázquez was supported by the Kempe foundation under Grant JCK-1239. The work of S. Aranovskiy was supported in part by the Government of Russian Federation under Grant 074-U01 and in part by the Ministry of Education and Science of Russian Federation under Project 14.Z50.31.0031. The work of L. M. Fridman was supported in part by the National Council of Science and Technology under Grant 261737 and in part by the Programa de Apoyo a Proyectos de Investigación e Innovación Tecnológica within the Universidad Nacional Autónoma de México, Grant 113612 and through the DGAPA PASPA Program. Recommended by Associate Editor E. Usai.

C. Vázquez is with Ålö AB, Brännland 300, Umeå SE-901 37, Sweden (e-mail: carlos.vazquez@alo.se).

S. Aranovskiy is with Inria, Non-A team, avenue Halley 40, Villeneuve d'Ascq 59650, France, and also with the Department of Control Systems and Informatics, ITMO University, Saint Petersburg 197101, Russia (e-mail: s.aranovskiy@gmail.com).

L. Freidovich is with the Robotics and Control Laboratory, Department of Applied Physics and Electronics, Umeå University, Umeå SE-901 87, Sweden (e-mail: leonid.freidovich@umu.se).

L. M. Fridman is with the Institut für Regelungs und Automatisierungstechnik, Graz 8010, Austria, on leave of the Departamento de Ingeniería de Control y Robótica, División de Ingeniería Eléctrica, Facultad de Ingeniería, Universidad Nacional Autonoma de Mexico, Ciudad Universitaria, Coyoacán 04510, Mexico.

Digital Object Identifier 10.1109/TCST.2015.2512880

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

such industrial activities as forestry, construction, agriculture, and mining, where high torques and a large ratio between the delivered force and the size of the actuator are required. Hydraulic actuators are able to provide large constant torques for a long period of time [22] that in the case of their electrical counterpart would cause overheating of motors. However, the system dynamics are characterized by strong nonlinearities, uncertainty in the parameters, as well as the presence of unknown perturbations. Traditionally in heavy-duty machines, these systems are controlled manually by a driver via a set of joysticks, and in a few cases, conventional linear feedback methods are integrated. In order to improve the efficiency, the driver's stress alleviation and to avoid accidents, a robust design of automatic control strategies are required. In contrast to industrial hydraulics, where high-precision sensors for pressure, position, velocity, and acceleration of the cylinders are available [23]–[25], in mobile hydraulics, the instrumentation is limited and includes only pressure transducers and low-accuracy position sensors. In addition, other nonlinear phenomena like dead zones and saturations are present, making the control design more difficult [26], [27]. In this case, the use of differentiators and observers has shown to increase the overall performance [28]–[31]; however, the optimal design for velocity estimation is an open subject.

In this note, a novel technique consisting of a first-order differentiator with a time-varying gain is proposed. The methodology combines two approaches: a high-gain observer with scaling-based analysis and a second-order sliding mode regime with exponential rate. Besides, a Lyapunov-function-based analysis is presented to demonstrate its properties, including the convergence rate and the ultimate boundedness of the differentiation error in the presence of noise. This provides a criterion for the enhancement of differentiator parameters under the presence of bounded noise. Experiments on a forestry-standard mobile hydraulic system have been carried out, obtaining very promising results. The comparison with others methods, including an offline estimation of the velocity using smoothing splines, confirms the efficacy of the methodology.

The remainder of this paper is organized as follows. First, a brief description of a first-order differentiator is presented in Section II. In Section III, we introduce the main contribution, which is the design of a time-varying gain differentiator (TVD) under the presence of bounded noise in the measurements. The case study and experimental results are presented in Section IV. Finally, in Section V, conclusions are drawn for this paper.

II. Problem Formulation: Differentiator

In mechanical systems, a first-order differentiator should estimate the first derivative of a position signal $x(t)$, under the presence of noise. Motivated by a particular application, we consider below the case when an upper bound of the second derivative is available for the differentiator in real time. The main assumptions are summarized as follows.

1) The second derivative of the position signal $x(t)$ is bounded by a continuous signal $L(t)$: $|\ddot{x}(t)| \leq L(t)$.
2) The signal $L(t)$ is bounded, $\underline{L} \leq L(t) \leq \bar{L}$.
3) The derivative of $L(t)$ is bounded: $|\dot{L}(t)| \leq L_d$, L_d is a known constant.

Defining $x_1 := x$ and $x_2 := \dot{x}$, the problem can be settled as the design of an observer for the system

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = \ddot{x}, \quad y = x + \upsilon \tag{1}$$

with the output y and bounded noise υ and assuming that $\ddot{x}$ can be described, for example, by an equation of motion originated from Newton's second law or by an Euler–Lagrange equation.

The output of an observer to be designed will be an estimate for the unmeasured state $x_2 = \dot{x}$.

Two approaches will be explored in order to solve this problem: second-order sliding modes and high-gain observers. The design is partially motivated by [21], where the inclusion of a time-varying gain, which depends on the available online signal $L(t)$, is the key idea.

Remark 1: In the case of constant gain, when some noise υ is present in the position measurement, i.e., $|\upsilon| \leq \upsilon_M$, for some $\upsilon_M > 0$, no on-line differentiator can provide for an accuracy of order better than $\mathcal{O}(\sqrt{L\,\upsilon_M})$ [3], [32], [33].

III. First-Order Time-Varying Gain Differentiator

In this section, a first-order differentiator with a time-varying gain is introduced. The algorithm is formed by the combination of two techniques: a high gain observer with time-varying scaled gains and a discontinuous term, understood in the equivalent control sense [34]. Furthermore, the algorithm includes a time-varying gain in order to increase performance and chattering attenuation as well as to separate the effects of disturbance attenuation and fast exponential convergence. The algorithm is as follows:

$$\begin{aligned} \dot{\hat{x}}_1 &= -\frac{\kappa_1}{\varepsilon} L_h^{\frac{1}{2}}(t)(\hat{x}_1 - y) + \hat{x}_2 \\ \dot{\hat{x}}_2 &= -\frac{\kappa_2}{\varepsilon^2} L_h(t)(\hat{x}_1 - y) - \underbrace{\kappa_3\, L(t)\, \mathrm{sign}(\hat{x}_1 - y)}_{\text{SM-term}} \end{aligned} \tag{2}$$

where κ_1, κ_2, and ε are positive constants, $L(t)$ satisfies Assumptions 1)–3) mentioned above, and although other choices are possible, we will simply take $L_h(t) = L(t)$ for the rest of this paper. Defining the scaled errors $e_1 = (\hat{x}_1 - x(t))\bar{L}^{-(1/2)}\varepsilon^{-1}$, $e_2 = (\hat{x}_2 - \dot{x}(t))\bar{L}^{-(1/2)}$, and $\mathbf{e} = [e_1, e_2]^T$, one obtains the perturbed equation

$$\varepsilon\, \dot{\mathbf{e}} = \mathbf{A}(t)\mathbf{e} + \varepsilon\, \mathbf{g}_0(t) + \varepsilon^{-1}\mathbf{g}_1(t)\upsilon(t) \tag{3}$$

where

$$\mathbf{A}(t) = \begin{bmatrix} -\kappa_1 L^{\frac{1}{2}}(t) & 1 \\ -\kappa_2 L(t) & 0 \end{bmatrix}, \quad \mathbf{g}_1(t) = \begin{bmatrix} k_1 L^{\frac{1}{2}}(t)\bar{L}^{-\frac{1}{2}} \\ -\kappa_2 L(t)\bar{L}^{-\frac{1}{2}} \end{bmatrix}$$

$$\mathbf{g}_0(t) = \begin{bmatrix} 0 \\ -\bar{L}^{-\frac{1}{2}}(\kappa_3\, L(t)\, \mathrm{sign}(e_1) + \ddot{x}(t)) \end{bmatrix}.$$

Let us start with an auxiliary statement to be used in the proofs in the following.

Proposition 1: Suppose positive constants α_1, κ_1, κ_2, $\underline{L}$, and $\bar{L}$ are such that the following inequalities are satisfied:

$$\alpha_1 > 1, \quad \underline{L} < \bar{L},$$
$$\kappa_2 \, \underline{L}^{\frac{1}{2}} \left(\alpha_1 \, \bar{L}^{\frac{1}{2}} - \underline{L}^{\frac{1}{2}}\right) - \frac{1}{4} \underline{q_{12}}^2 > 0 \quad (4)$$

where $\underline{q_{12}} = \kappa_1 \, \underline{L}^{(1/2)}((\underline{L}^{(1/2)}/\bar{L}^{(1/2)}) - 1) + \alpha_1(\kappa_2/\kappa_1)\bar{L}^{(1/2)}((\underline{L}/\bar{L}) - 1)$. Then, the next matrices are positive definite

$$\mathbf{P} = \begin{bmatrix} \left(\frac{\alpha_1 \kappa_2}{\kappa_1}\right) \bar{L}^{\frac{1}{2}} & -1 \\ -1 & \left(\frac{\alpha_1 \kappa_2 + \kappa_1^2}{\kappa_1 \kappa_2}\right) \frac{1}{\bar{L}^{\frac{1}{2}}} \end{bmatrix}$$

$$\underline{\mathbf{Q}} = \begin{bmatrix} \kappa_2 \, \underline{L}^{\frac{1}{2}} \left(\alpha_1 \, \bar{L}^{\frac{1}{2}} - \underline{L}^{\frac{1}{2}}\right) & \frac{1}{2} \underline{q_{12}} \\ \frac{1}{2} \underline{q_{12}} & 1 \end{bmatrix}$$

Proof: Note that using Sylvester's criterion, Proposition 1 can be easily verified. ■

In the following and in the rest of this paper, $\lambda_{\max(\min)}[\cdot]$ denotes the operation of taking the largest (smallest) eigenvalue of some symmetric matrices and that inequalities between matrices are componentwise. The Euclidean norm of a vector $\mathbf{x}$ and the induced norm of a matrix $\mathbf{A}$ are denoted by $\|\mathbf{x}\|$ and $\|\mathbf{A}\|$, respectively. On the other hand, the lower and upper constant bounds, elementwise, for a time-varying matrix, $\mathbf{A}(t)$, are given by $\underline{\mathbf{A}}$ and $\bar{\mathbf{A}}$, respectively.

Proposition 2: Consider the scaled error dynamics (3), where $\ddot{x}(t) \leq L(t)$ and $\underline{L} \leq L(t) \leq \bar{L}$. There exist matrices $\mathbf{P}$ and $\underline{\mathbf{Q}}$ as in Proposition 1 such that for all bounded noise signals υ, the solutions $\mathbf{e}(t)$ are globally uniformly ultimately bounded by $(1/\underline{\delta})((\bar{\varrho}/\underline{\varrho}))\bar{\mu}(\varepsilon, \upsilon)$, where

$$\bar{\mu}(\varepsilon, \upsilon) = (\varepsilon\|\mathbf{g}_0\|_\infty + \varepsilon^{-1}\|\mathbf{g}_1\|_\infty \, \|\upsilon\|_\infty)\bar{\varrho}$$

with $\underline{\varrho} = \lambda_{\min}[\mathbf{P}]$, $\bar{\varrho} = \lambda_{\max}[\mathbf{P}]$, $\underline{\delta} = \lambda_{\min}[\underline{\mathbf{Q}}]$, and ε could be arbitrary small, i.e., there is a positive constant T such that

$$\|\mathbf{e}(t)\| \leq \frac{1}{\underline{\delta}} \left(\frac{\bar{\varrho}}{\underline{\varrho}}\right) \bar{\mu}(\varepsilon, \upsilon) \quad \forall t \geq T. \quad (5)$$

Proof: Consider the Lyapunov function

$$V(\mathbf{e}) = \frac{\varepsilon}{2} \mathbf{e}^T \mathbf{P} \mathbf{e} \quad (6)$$

where $\mathbf{P}$ is a positive definite symmetric constant matrix, given in Proposition 1, which implies

$$\frac{\varepsilon}{2} \underline{\varrho} \, \|\mathbf{e}\|^2 \leq V \leq \frac{\varepsilon}{2} \bar{\varrho} \, \|\mathbf{e}\|^2 \quad (7)$$

where $\underline{\varrho} = \lambda_{\min}[\mathbf{P}]$ and $\bar{\varrho} = \lambda_{\max}[\mathbf{P}]$.

Now, taking the derivative of V along (3), one obtains

$$\dot{V} = -\mathbf{e}^T \mathbf{Q}(t)\mathbf{e} + \left(\varepsilon \mathbf{g}_0(t) + \frac{1}{\varepsilon}\mathbf{g}_1(t)\upsilon(t)\right) \mathbf{P}\mathbf{e} \quad (8)$$

where $\mathbf{P}$ and $\mathbf{Q}(t)$ are related by the following Lyapunov equation:

$$\mathbf{A}^T(t)\mathbf{P} + \mathbf{P}\mathbf{A}(t) = -2\mathbf{Q}(t). \quad (9)$$

In order to guarantee stability, we are looking for solutions of (9) satisfying the next additional assumption

$$0 < \underline{\mathbf{Q}} \leq \mathbf{Q}(t) \leq \bar{\mathbf{Q}}. \quad (10)$$

With the particular selection for the matrix $\mathbf{P}$, we have

$$\mathbf{Q}(t) = \begin{bmatrix} \kappa_2 \, L^{\frac{1}{2}}(t)\left(\alpha_1 \, \bar{L}^{\frac{1}{2}} - L^{\frac{1}{2}}(t)\right) & \frac{1}{2} q_{12}(t) \\ \frac{1}{2} q_{12}(t) & 1 \end{bmatrix}$$

where $q_{12}(t) = \kappa_1 \, L^{(1/2)}(t)((L^{(1/2)}(t)/\bar{L}^{(1/2)}) - 1) + \alpha_1(\kappa_2/\kappa_1)\bar{L}^{(1/2)}((L(t)/\bar{L}) - 1)$ and $\underline{\mathbf{Q}}$ is given in Proposition 1. It is not hard to verify that with the previous assumptions

$$\mathbf{e}^T \mathbf{Q}(t)\mathbf{e} \geq \underline{\delta}\|\mathbf{e}\|^2$$

with $\underline{\delta} = \lambda_{\min}[\underline{\mathbf{Q}}] > 0$. With the following change of the auxiliary function $W_1^2 = V$, we have:

$$\varepsilon \dot{W}_1 \leq -\underline{\mu_0}(\varepsilon)W_1 + \frac{1}{2^{\frac{1}{2}}} \varepsilon^{\frac{1}{2}} \, \underline{\varrho}^{-\frac{1}{2}} \, \bar{\mu}(\varepsilon, \upsilon)$$

where $\bar{\mu}(\varepsilon, \upsilon) = (\varepsilon\|\mathbf{g}_0\|_\infty + \varepsilon^{-1}\|\mathbf{g}_1\|_\infty \, \|\upsilon\|_\infty)\bar{\varrho}$ and $\underline{\mu_0}(\varepsilon) = \underline{\delta} / \bar{\varrho}$. Hence, there exists an instant of time T such that $\forall t \geq T$

$$W_1 \leq \frac{1}{2^{\frac{1}{2}}} \varepsilon^{\frac{1}{2}} \frac{\bar{\varrho}}{\underline{\delta} \, \underline{\varrho}^{\frac{1}{2}}} \, \bar{\mu}(\varepsilon, \upsilon). \quad (11)$$

Finally, from (7) and (11), we obtain the desired result

$$\|\mathbf{e}\| \leq \left(\frac{2}{\varepsilon \, \underline{\varrho}}\right)^{\frac{1}{2}} W_1 \leq \frac{1}{\underline{\delta}} \left(\frac{\bar{\varrho}}{\underline{\varrho}}\right) \bar{\mu}(\varepsilon, \upsilon).$$

■

Remark 2: In general, an ultimate bound of steady-state errors, obtained from the Lyapunov method, is very conservative; however, it can give us valuable information for a constructive selection of the parameters κ_1, κ_2, and ε.

Now, we are going to illustrate how the constants κ_1 and κ_2 can be computed for a given gain $L(t)$ and for a particular magnitude of noise υ. Solving inequalities (4) and (5), different values for the ultimate bound (5) can be obtained. To simplify this, let us define the level sets d_i, for $i = 0, 1, \ldots, 6$, such that $\|\mathbf{e}\| \leq d_i$; here, each subset d_i represents a particular upper bound of the estimation error (5). For experimental purposes, let us consider $\alpha_1 = 4$, $\underline{L} = 5$, $\bar{L} = 70$, $\varepsilon = 0.007$, and $||\upsilon||_\infty = 0.001$. With this information, the feasible solutions for the pair (κ_1, κ_2) are obtained for each predefined level set d_i. This is done with a numerical procedure resulting in the picture presented in Fig. 1, which represents the stability region and provides an optimal criterion for the selection of the pair (κ_1, κ_2) for a particular ε and an amplitude of noise, in this case $||\upsilon||_\infty = 0.001$. In addition, it is interesting to see how the value of the ultimate bound on the estimation error is affected by the selection of ε. For this aim, let us fix a pair (κ_1, κ_2) from each level set d_i in the stability region (see Fig. 1). Then, inequality (4) is computed as a function of ε, i.e., $\|\mathbf{e}\|$ versus ε, as shown in Fig. 2.

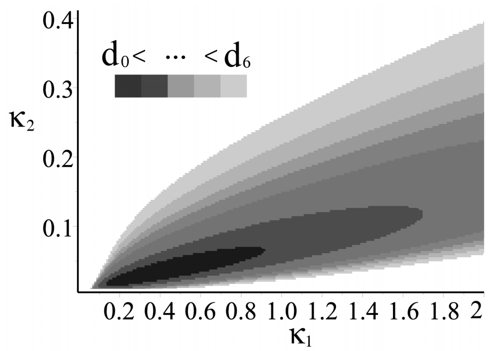

Fig. 1. Stability region, κ_2 versus κ_1, with $\varepsilon = 0.007$, $\alpha_1 = 4$, $\underline{L} = 5$, $\bar{L} = 70$, and $||v||_\infty = 0.001$.

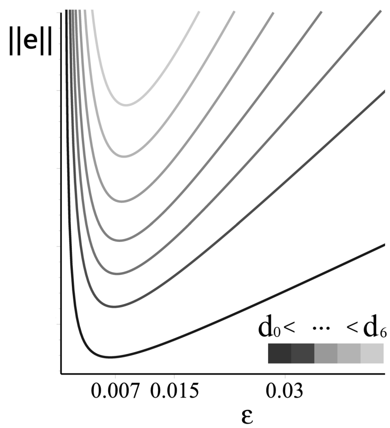

Fig. 2. Ultimate bound estimation, $\|\mathbf{e}\|$ versus ε, for different level sets, $\alpha_1 = 4$, $\underline{L} = 5$, $\bar{L} = 70$, and $||v||_\infty = 0.001$.

A. Exponential Convergence

In the proposition 2, it has been shown that the proposed differentiator provides ultimate bounded convergence under the presence of noise. In contrast, in this section, it is shown that in the absence of noise, the differentiator provides an exponential convergence rate of estimation, which is an important property. For this purpose, let us consider $y = x$, i.e., $v = 0$. Now, defining the scaled errors differently as $e_1 = (\hat{x}_1 - x(t))\varepsilon^{-1}$, $e_2 = -\kappa_1 L^{(1/2)} e_1 + \hat{x}_2 - \dot{x}(t)$, and $\boldsymbol{e} = [e_1, e_2]^T$, one obtains the perturbed equation

$$\varepsilon\,\dot{\mathbf{e}} = \mathbf{f}(t, \mathbf{e}) + \varepsilon\,\mathbf{g}_0(t) \tag{12}$$

with $\mathbf{f}(t, \mathbf{e}) = (\mathbf{A}(t) + \varepsilon\,\Delta\mathbf{A}(t))\mathbf{e}$, where

$$\mathbf{A}(t) = \begin{bmatrix} 0 & 1 \\ -\kappa_2 L & -\kappa_1 L^{\frac{1}{2}} \end{bmatrix}, \quad \Delta\mathbf{A}(t) = \begin{bmatrix} 0 & 0 \\ -\dfrac{\kappa_1 \dot{L}}{2 L^{\frac{1}{2}}} & 0 \end{bmatrix}$$

$$\mathbf{g}_0(t) = \begin{bmatrix} 0 \\ -\kappa_3 L\ \mathrm{sign}(e_1) - \ddot{x}(t) \end{bmatrix}.$$

Note that the perturbation term $\ddot{x}(t)$ is still present.

Remark 3: A necessary condition for existence of solutions in either the Filippov [35] or the equivalent control sense [34] as well as possibility for the appearance of the asymptotic sliding mode is, obviously, $\kappa_3 > 1$.

Proposition 3: Consider the time-varying matrices

$$\mathbf{P}(t) = \begin{bmatrix} p_{11}(t) & p_{12} \\ p_{12} & p_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{\kappa_2\,\alpha\,L(t)}{\kappa_1\,\underline{L}^{\frac{1}{2}}} + \kappa_1\,L^{\frac{1}{2}}(t) & 1 \\ 1 & \dfrac{\alpha}{\kappa_1\,\underline{L}^{\frac{1}{2}}} \end{bmatrix} \tag{13}$$

and

$$\mathbf{Q}(t) = \begin{bmatrix} \kappa_2\,L(t) & 0 \\ 0 & \alpha - 1 \end{bmatrix} \tag{14}$$

where α, κ_1, κ_2, $\underline{L}$ and $\bar{L}$ are positive constants, and $\underline{L} \le L(t) \le \bar{L}$. Choosing $\alpha > 1$, we have $\mathbf{Q}(t) > 0$ and $\mathbf{P}(t) > 0$, satisfying the bounds $\underline{\mathbf{P}} \le \mathbf{P}(t) \le \bar{\mathbf{P}}$ and $\underline{\mathbf{Q}} \le \mathbf{Q}(t) \le \bar{\mathbf{Q}}$, where

$$\underline{\mathbf{P}} = \begin{bmatrix} \left(\dfrac{\kappa_2\,\alpha}{\kappa_1} + \kappa_1\right)\underline{L}^{\frac{1}{2}} & 1 \\ 1 & \dfrac{\alpha}{\kappa_1\,\underline{L}^{\frac{1}{2}}} \end{bmatrix}$$

$$\bar{\mathbf{P}} = \begin{bmatrix} \left(\dfrac{\kappa_2\,\alpha\,\bar{L}^{\frac{1}{2}}}{\kappa_1\,\underline{L}^{\frac{1}{2}}} + \kappa_1\right)\bar{L}^{\frac{1}{2}} & 1 \\ 1 & \dfrac{\alpha}{\kappa_1\,\underline{L}^{\frac{1}{2}}} \end{bmatrix}$$

$$\underline{\mathbf{Q}} = \begin{bmatrix} \kappa_2\,\underline{L} & 0 \\ 0 & \alpha - 1 \end{bmatrix}, \quad \bar{\mathbf{Q}} = \begin{bmatrix} \kappa_2\,\underline{L} & 0 \\ 0 & \alpha - 1 \end{bmatrix}.$$

Proposition 3 follows from Sylvester's criterion.

Proposition 4: Consider error dynamics (12) with $0 < \underline{L} \le L(t) \le \bar{L}$, $|\ddot{x}| \le L(t)$, and $\dot{L}(t) \le L_d$; there exist matrices $\mathbf{P}(t)$ and $\mathbf{Q}(t)$ defined as in Proposition 3 such that provided the conditions

$$L_d < \min\left\{\frac{1}{\varepsilon}\frac{\kappa_3 - 1}{\kappa_3 + \kappa_4}\underline{L}, \frac{\lambda_{\min}[\underline{\mathbf{Q}}]}{\varepsilon\,\lambda_{\max}[\bar{\mathbf{B}}]}\right\}$$

$$\min\left\{\frac{1}{2}(\lambda_{\min}[\underline{\mathbf{Q}}] - \varepsilon\lambda_{\max}[\bar{\mathbf{B}}]L_d), \varepsilon\min\{q_0, q_1\}\,p_{22}\right\} > 0$$

$$1 < \kappa_4 \le \kappa_3$$

with $q_0 = \kappa_4\,L - \ddot{x}\ \mathrm{sign}(e_2)$ and $q_1 = \kappa_3 L + \varepsilon(\kappa_3 - \kappa_4\,\mathrm{sign}(e_1 e_2))(d/dt)(p_{22}L)p_{22}^{-1} + \ddot{x}\ \mathrm{sign}(e_1)$ and

$$\bar{\mathbf{B}} = \begin{bmatrix} \left(\dfrac{\alpha\,\kappa_2}{\kappa_1} - \dfrac{\kappa_1}{2}\right)\dfrac{1}{\underline{L}^{\frac{1}{2}}} & -\dfrac{\alpha}{2\underline{L}} \\ -\dfrac{\alpha}{2\underline{L}} & 0 \end{bmatrix}$$

being satisfied, the equilibrium $\mathbf{e} = 0$ is globally exponentially stable.

Proof: Consider the discontinuous Lyapunov function

$$V(t, \mathbf{e}) = \frac{\varepsilon}{2}\mathbf{e}^T\mathbf{P}(t)\mathbf{e} + \varepsilon^2\,p_{22}L|e_1|(\kappa_3 - \kappa_4\,\mathrm{sign}(e_1\,e_2)) \tag{15}$$

where κ_4 is a positive constant satisfying $1 < \kappa_4 \le \kappa_3$ and $\mathbf{P}(t)$ is a positive definite symmetric matrix containing time-dependent elements on the diagonal as in Proposition 3. In particular, the elements of the diagonal are continuous function of $L(t)$. Monotonicity of V will be proved using the generalized contingent derivatives (see the Appendix for more details or [36]–[38] for a complete study). Moreover, under Proposition 3, the next properties for matrix $\mathbf{P}(t)$ are ensured

$$\begin{aligned} &0 < \underline{\mathbf{P}} \le \mathbf{P}(t) \le \bar{\mathbf{P}} \\ &\Delta\mathbf{A}^T\mathbf{P}(t) + \mathbf{P}(t)\Delta\mathbf{A} \triangleq \mathbf{B}_0 \frac{dL}{dt} \\ &\frac{d}{dt}\mathbf{P}(t) = \mathbf{B}_1 \frac{dL}{dt}, \quad \text{with } \mathbf{B}_1 \triangleq \frac{\partial}{\partial L}\mathbf{P} \\ &0 < \underline{\mathbf{B}} \le \mathbf{B}(t) \le \bar{\mathbf{B}}, \ \text{with } \mathbf{B}(t) = \mathbf{B}_0 + \mathbf{B}_1. \end{aligned} \tag{16}$$

In addition

$$\varepsilon\,\underline{\varphi}\,\|\mathbf{e}\|^2 \le V \le \varepsilon\,\bar{\varphi}\,(\|\mathbf{e}\|^2 + \|\mathbf{e}\|) \tag{17}$$

where $\underline{\varphi} = \min\{(1/2)\lambda_{\min}[\underline{\mathbf{P}}], \varepsilon p_{22}\underline{L}(\kappa_3 + \kappa_4)\}$ and $\bar{\varphi} = \max\{(1/2)\lambda_{\max}[\bar{\mathbf{P}}], \varepsilon p_{22}\bar{L}(\kappa_3 + \kappa_4)\}$. The Lyapunov function V is globally proper, Lipschitz continuous outside the origin, and continuously differentiable for $e_1 e_2 \neq 0$. Here, we need to make use of the generalized derivatives introduced in the Appendix. In particular, for $e_1 e_2 \neq 0$, $D_F V = (\partial/\partial t)V + (\partial/\partial \mathbf{e})V(\varepsilon^{-1} f(t, \mathbf{e}) + \mathbf{g}_0(t))$. Then, taking the generalized derivative, $D_F V$, along (3) for $e_1 e_2 \neq 0$

$$D_F V = -W(\mathbf{e}, t) \tag{18}$$

where $W(\mathbf{e}, t) = (1/2)\mathbf{e}^T\mathbf{Q}_0\mathbf{e} + \varepsilon q_0 p_{22}|e_2| + \varepsilon q_1 p_{22}|e_1|$, with $\mathbf{Q}_0 = \mathbf{Q} + \varepsilon\,\mathbf{B}\dot{L}$, $q_0 = \kappa_4 L - \ddot{x}\,\operatorname{sign}(e_2)$, and $q_1 = \kappa_3 L + \varepsilon(\kappa_3 - \kappa_4 \operatorname{sign}(e_1 e_2))(d/dt)(p_{22}L)p_{22}^{-1} + \ddot{x}\,\operatorname{sign}(e_1)$, with $L_d < (1/\varepsilon)(\kappa_3 - 1/\kappa_3 + \kappa_4)\underline{L}$. It is not hard to verify that with the previous assumptions, the next relation holds

$$\underline{\eta}(\|\mathbf{e}\|^2 + \|\mathbf{e}\|) \le W(\mathbf{e}, t) \le \bar{\eta}(\|\mathbf{e}\|^2 + \|\mathbf{e}\|) \tag{19}$$

where

$$\underline{\eta} = \min\left\{\frac{1}{2}(\lambda_{\min}[\underline{\mathbf{Q}}] - \varepsilon\,\lambda_{\max}[\bar{\mathbf{B}}]L_d),\ \varepsilon\,\min\{q_0, q_1\}\,p_{22}\right\}$$

$$\bar{\eta} = \max\left\{\frac{1}{2}(\lambda_{\max}[\bar{\mathbf{Q}}] - \varepsilon\,\lambda_{\min}[\bar{\mathbf{B}}]L_d),\ \varepsilon\,\max\{q_0, q_1\}\,p_{22}\right\}.$$

Considering (18) together with (17) and (19), we obtained the desired result

$$-\frac{\bar{\eta}}{\varepsilon\underline{\varphi}}\left(1 + \varepsilon^{\frac{1}{2}}\underline{\varphi}^{\frac{1}{2}}V^{-\frac{1}{2}}\right)V \le D_F V \le -\frac{\underline{\eta}}{\varepsilon\bar{\varphi}}\,V.$$

It is important to remark that the function V is discontinuous on the line $e_2 = 0$ for $e_1 \neq 0$, where $D_F V(e_1, 0) = \{-\infty\}$ for $e_1 \neq 0$ and $D_F V(0, e_2) \le -(\underline{\eta}/\varepsilon\bar{\varphi})\,V(0, e_2)$ for $e_2 \neq 0$. Note that these generalized derivatives can be obtained with the help of Dini derivatives, as is shown in the Appendix. ∎

Setting particular values for ε and L_d, it is straightforward to verify the required conditions in Proposition 4. Fig. 3 shows the stability region in terms of κ_1 and κ_2 for $\varepsilon = 0.007$, $\alpha = 4$, $\underline{L} = 5$, $\bar{L} = 70$, $L_d = 40$, $\kappa_3 = 1.2$, and $\kappa_4 = 1.1$. We would like to point out that the obtained upper bound for L_d, i.e., $L_d \le (1/\varepsilon)K(\kappa_1, \kappa_2)$, where $K(\kappa_1, \kappa_2) = \min\{(\kappa_3 - 1/\kappa_3 + \kappa_4)\underline{L}, (\lambda_{\min}[\underline{\mathbf{Q}}]/\lambda_{\max}[\bar{\mathbf{B}}])\}$, is due to the proposed Lyapunov function. The design of a Lyapunov function for an arbitrary L_d is an open problem even for the linear case. However, the proposed Lyapunov function offers a tradeoff between the selection of ε and L_d: the lower the value of ε is taken, the greater the value of L_d can be tolerated.

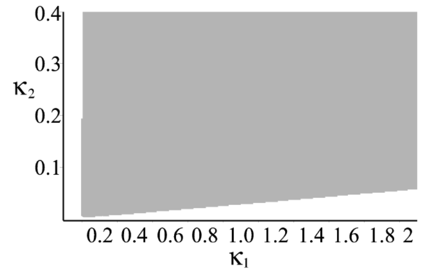

Fig. 3. Stability region, κ_1 versus κ_2, for $\varepsilon = 0.007$, $\alpha = 4$, $\underline{L} = 5$, $\bar{L} = 70$, $L_d = 40$, $\kappa_3 = 1.2$, and $\kappa_4 = 1.1$.

TABLE I
PHYSICAL PARAMETERS OF THE LINK

A_a, m^2	A_b, m^2	V_{a0}, m^3	V_{b0}, m^3
$1.26 \cdot 10^{-3}$	$0.76 \cdot 10^{-3}$	$0.012 \cdot 10^{-3}$	$1.19 \cdot 10^{-3}$
P_t, Pa	P_s, Pa	$\bar{q}$, l/min	β, Pa
$5 \cdot 10^5$	$180 \cdot 10^5$	90	$17 \cdot 10^8$
m, kg	$\bar{f}_h$, N	f_c, N	f_v, N·s/m
200	8000	750	6500

IV. MOBILE HYDRAULIC SYSTEM CASE STUDY

The experimental setup under the study is a laboratory prototype of an industry-standard hydraulic forestry crane. Such equipment is widely used in forestry and is a subject of many research studies aimed at automation or remote monitoring of such systems [39]. One important related issue is the online velocity estimation problem. In this section, we show how this problem can be successfully solved by the proposed differentiator. We solve this problem for a telescopic link of the crane; however, similar results can be easily obtained for the other joints. Some physical parameters of the link are given in Table I.

A. Modeling Mechanical and Hydraulic Systems

The telescopic link of the crane consists of a double-acting single–side hydraulic cylinder and a solid load, which is attached to a piston of the cylinder (see Fig. 4).

The position of the link x varies from 0 to 1.55 m; positive velocity $\dot{x} > 0$ corresponds to extraction of the cylinder. This link can be described as a 1-DOF mechanical system actuated by a hydraulic force, and the equation of the motion is

$$m\ddot{x} = f_h - f_{\text{grav}} - f_{\text{fric}}$$

where m is the mass, f_h is the force generated by the hydraulics, f_{grav} is the gravity force, and f_{fric} is the friction force. The force generated by the hydraulics is given by

$$f_h = P_a A_a - P_b A_b \tag{20}$$

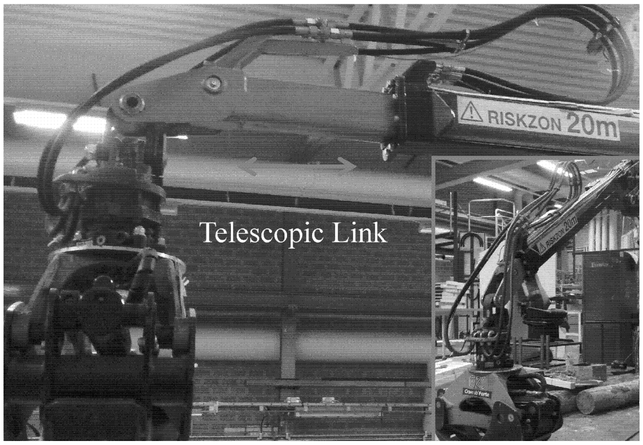

Fig. 4. Industrial hydraulic forestry crane.

where the piston areas A_a and A_b are known geometric parameters, while P_a and P_b are the measured pressures in chambers A and B of the cylinder, respectively. The friction is approximated by Coulomb and viscous models: $f_{\text{fric}} = f_c \operatorname{sign}(\dot{x}) + f_v \dot{x}$. Hence

$$\begin{aligned} \ddot{x} &= \frac{f_h}{m} - \frac{1}{m}(f_{\text{grav}} + f_c \operatorname{sign}(\dot{x})) - \frac{f_v}{m}\dot{x} \\ &= \frac{f_h}{m} - f_0 - f_1 \dot{x}. \end{aligned} \tag{21}$$

During normal operation, the dynamics of the pressures can be approximately described [22, Sec. 3.8] by

$$\dot{P}_a = \frac{\beta}{V_a(x)}(-\dot{x} A_a + q_a), \quad \dot{P}_b = \frac{\beta}{V_b(x)}(\dot{x} A_b - q_b) \tag{22}$$

where $V_a(x) = V_{a0} + x A_a$ and $V_b(x) = V_{b0} - x A_b$ are volumes of the chambers A and B for the given piston position x, respectively, V_{a0} and V_{b0} are known geometric constants, β is an experimentally identified bulk modulus, and q_a and q_b are flows to the chamber A and from the chamber B, respectively. Differentiating (20) and substituting (22) lead to

$$\begin{aligned} \dot{f}_h &= \frac{\beta}{V_a(x)V_b(x)}(A_a V_b(x)q_a + A_b V_a(x)q_b) \\ &\quad - \frac{\beta}{V_a(x)V_b(x)}(A_a^2 V_b(x) + A_b^2 V_a(x))\dot{x}. \end{aligned}$$

Therefore, we obtain $\dot{x} = \eta_0(x, q_a, q_b) - \eta_1(x)\dot{f}_h$, where

$$\begin{aligned} \eta_0(x, q_a, q_b) &= \frac{A_a V_b(x)q_a + A_b V_a(x)q_b}{A_a^2 V_b(x) + A_b^2 V_a(x)} \\ \eta_1(x) &= \frac{V_a(x)V_b(x)\beta^{-1}}{A_a^2 V_b(x) + A_b^2 V_a(x)}. \end{aligned} \tag{23}$$

From (21), the following expression is obtained:

$$\begin{aligned} \ddot{x} &= \frac{1}{m} f_h - f_0 - f_1 \eta_0(x, q_a, q_b) + f_1 \eta_1(x)\dot{f}_h \\ &= \frac{f_h}{m} - c_0(x, q_a, q_b) + c_1(x)\dot{f}_h. \end{aligned} \tag{24}$$

B. Bounds on the Variables

Both pressures P_a and P_b are bounded by the tank pressure P_t and the supply pressure P_s. However, it is not a realistic practical situation when both pressures have extreme contrary values simultaneously. Due to internal restrictions, the practical bound is $|f_h| \le \bar{f}_h$. The bound for the gravity force $\bar{f}_g$ is defined by the given mass. Hence, for (21), we can define the upper bound $|f_0| \le \bar{f}_0$ with $\bar{f}_0 = (\bar{f}_g + f_c)/m$. The parameter $f_1 = f_v/m > 0$ is constant.

Both flows q_a and q_b are bounded by a factory-set level of a maximum flow through a valve, $|q_{a,b}| \le \bar{q}$ [26]. Moreover, the flows cannot go in the same direction simultaneously, i.e., they are always of the same sign. Hence, upper bounds $|\eta_0(x, q_a, q_b)| \le \bar{\eta}_0$ and $|\eta_1(x)| \le \bar{\eta}_1$ can be defined for (23), and functions $c_0(x, q_a, q_b)$ and $c_1(x)$ in (24) are bounded by $|c_0(x, q_a, q_b)| \le \bar{c}_0$ and $|c_1(x)| \le \bar{c}_1$, where $\bar{c}_0 = \bar{f}_0 + f_1 \bar{\eta}_0$ and $\bar{c}_1 = f_1 \bar{\eta}_1$.

A practical (experimentally found) bound on the velocity is $|\dot{x}| \le \bar{x}^{(1)}$ with $\bar{x}^{(1)} = 1.1\,\text{m/s}$. It follows from (21) that the acceleration $\ddot{x}$ is bounded by $|\ddot{x}| \le \bar{x}^{(2)}$, where:

$$\bar{x}^{(2)} = \frac{1}{m}\bar{f}_h + \bar{f}_0 + f_1 \bar{x}^{(1)}.$$

As the flows q_a and q_b are bounded, it follows from (22) that both time derivatives $\dot{P}_a$ and $\dot{P}_b$ are bounded $|\dot{P}_i| \le (\beta/V_{i0})(A_i\bar{x}^{(2)} + \bar{q})$, $i = a, b$. This implies $|\dot{f}_h| \le |\dot{P}_a|A_a + |\dot{P}_b|A_b \le c_2$.

C. Measured and Estimated Signals

The experimental tests are carried out with a real-time platform dSpace 1401 at a sampling interval of 1 (ms) using the forward Euler integration method. The pressures are measured with installed pressure transducers that allow us to estimate the force (20) that later will be used to design the profile $L(t)$. The position of the telescopic link is measured with a wire-actuated encoder. The encoder provides 2381 counts for the range from 0 to 1.55 m and the quantization interval is $Q = 0.651$ mm. Such a quantization interval makes it hard to use a direct difference of the position for velocity estimation as the resulting velocity quantization interval is inappropriately high.

The differentiators considered in this paper are designed to be used online. However, it is obvious that a better velocity estimation can be achieved with an offline method when both previous and future values of the position are used. Based on this idea, we suppose postprocessing the measured position with an offline velocity estimation method to obtain an estimation $\hat{x}_{2,\text{off}}$. Further, we evaluate the designed online differentiators in comparison with this offline estimation.[1]

To obtain the offline estimation, we use splines. First, the measured signal $x(t)$ is fitted with a smoothing spline $x_{\text{spl}}(t)$. Next, $\hat{x}_{2,\text{off}}$ is obtained as an analytical differentiation of

[1]We would like to highlight that the obtained offline estimation is not considered as a real velocity, which is not possible to be measured in this experiment; the proposed offline estimation is used only as a baseline for further evaluations. However, multiple studies carried out with a numerical model of the forestry crane dynamics show that the proposed spline-based offline velocity estimation gives a good approximation of the real velocity.

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

the spline $x_{\rm spl}(t)$. The smoothing spline $x_{\rm spl}(t)$ is found as a cubic spline that minimizes the following expression (see [40, p. 194]):

$$\rho \sum_{i=1}^{N} (x(t_i) - x_{\rm spl}(t_i))^2 + (1-\rho) \int_{t_1}^{t_N} \ddot{x}_{\rm spl}^2 \, dt$$

where N is the number of measured points and $0 \leq \rho \leq 1$ is a smoothing parameter. The smoothing parameter determines a tradeoff between fitting of the measured data and smoothing. The value $\rho = 0$ leads to maximum smoothing, i.e., linear approximation, and $\rho = 1$ leads to a classic cubic spline with exact fitting and without any smoothing. For our purposes, we tune the smoothing parameter in order to obtain the smoothest possible estimation, i.e., the smallest ρ, keeping the fitting error within the quantization error Q.

D. Design of L(t)

In order to use our proposed solution for the problem of obtaining the velocity of the spool of the cylinder from the measured position, we need a reliable estimate on the bound for the acceleration. In the other words, to use the proposed differentiator (2), we need to design some appropriated time-varying gain $L(t)$, such that $|\ddot{x}| \leq L(t)$ and $|\dot{L}| \leq \delta_1$. Besides, from (24), the following bound is obtained:

$$|\ddot{x}| \leq \bar{c}_0 + \frac{1}{m}|f_h| + \bar{c}_1|\dot{f}_h|. \tag{25}$$

When the cylinder is moving with a constant velocity, we have $\ddot{x} \approx 0$, which means that a small constant gain L can be selected. Besides, if the cylinder is moving with varying velocity, then the acceleration $\ddot{x}(t)$ is not close to zero anymore and it implies that the gain L should increase proportionally to the rate of variation of the cylinder velocity. In order to include both cases, constant and time-varying profiles of velocity, and motivated by the expressions (24) and (25), we proposed the next time-varying gain

$$L(t) = \gamma_0 + \gamma_1|f_h| + \gamma_2\zeta(f_h) \tag{26}$$

where the parameters γ_0, γ_1, and γ_2 are positive constants, and $\zeta(f_h)$ represents an upper bound of the rate of variation of f_h, particularly, and $\zeta(f_h)$ is a positive function that depends on the available pressure measurements. In order to construct such an upper bound, the exact derivative of f_h is not needed, and either a linear observer or a filter can be used for this purpose. One of the options, which have been successfully tested and keep a simple structure, is the following:

$$\zeta(f_h) = \frac{|f_h(t-\tau_1) - f_h(t-\tau_2)|}{\tau_2 - \tau_1}$$

where $\tau_2 > \tau_1 > 0$. With this selection, the upper bound of the derivative $\dot{L}$ is given by $|\dot{L}| \leq \gamma_1 c_2 + (2\gamma_2 c_2/\tau_2 - \tau_1)$.

Four velocity estimation algorithms are tested.

1) The supertwisting differentiator with constant gain: STA[2] [6].

[2]We consider the STA algorithm in the original form

$$\dot{\hat{x}}_1 = -1.5\, L^{\frac{1}{2}} |\hat{x}_1 - x(t)|^{\frac{1}{2}} \, \mathrm{sign}(\hat{x}_1 - x(t)) + \hat{x}_2$$
$$\dot{\hat{x}}_2 = -1.1\, L \, \mathrm{sign}(\hat{x}_1 - x(t)).$$

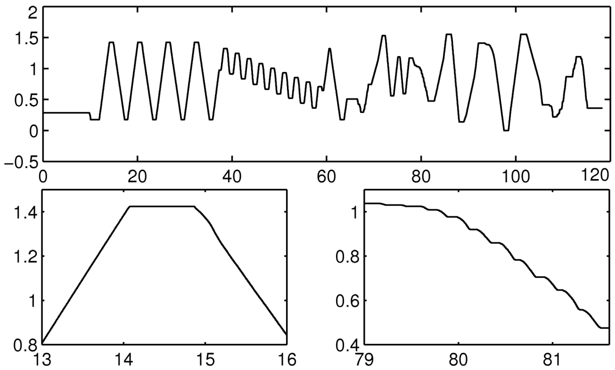

Fig. 5. Measured position x (in meters) versus time (in seconds).

2) The STA with time-varying gain: STAV[3] [21].
3) The proposed TVD (2).
4) The proposed TVD with a constant gain, the algorithm (2) with L as a constant: CD (Constant Gain Differentiator).

E. Selection of the Differentiator Parameters

There are two important elements to be designed: $L(t)$ and ε. For the design of $L(t)$, we use the proposed time-varying gain given by (26), with the corresponding parameters: $\gamma_0 = 5$, $\gamma_1 = 0.0003$, $\gamma_2 = 0.00035$, $\tau_1 = 0.004$, and $\tau_2 = 0.01$.

Any profile of acceleration can be covered with this $L(t)$ ($\underline{L} = 5$, $\bar{L} = 70$), as can be verified from Figs. 6, 7, and 9. The design for $L(t)$ is not unique and we can modify the involved parameters from a comparison with the offline estimation of acceleration.

The measured signal x can be seen as the position signal with an additive uniform noise with a variance $Q^2/12$, which implies $\|v\|_\infty \leq 0.001$. In this case, the values of ε, κ_1 and κ_2 can be selected in an optimal way using Figs. 1 and 2 from the previous Lyapunov analysis. From Figs. 1 and 2, we select the parameters $\kappa_1 = 0.4$, $\kappa_2 = 0.03$, and $\varepsilon = 0.007$. In addition, for the proposed algorithm, we have fixed $\kappa_3 = 1.1$.

For evaluation purposes, we compare all the algorithms with the velocity estimation obtained using postprocessing of the measured data, i.e., offline velocity estimation (see Section IV-C). In the experiment, we consider different input profiles and Fig. 5 shows the measured cylinder position x. In this experiment, the cylinder was in motion with constant and varying velocity.

F. Super-Twisting Algorithm: Time-Varying Versus Constant Gain

In this section, we compared the STA using constant and time-varying gains. The STAV differentiator is implemented considering the time-varying gain (26). For the gain $L(t)$, we take $\gamma_0 = 5$, $\gamma_1 = 0.0003$, $\gamma_2 = 0.00035$, $\tau_1 = 0.004$, and $\tau_2 = 0.01$. The sign function is approximated by $\mathrm{sign}(x) = (x/|x| + \epsilon)$, with $\epsilon = 0.001$[4]. For comparison purposes,

[3]The STAV preserves the same structure with a time-varying gain, $L = L(t)$.

[4]Due to the sampling time and the limited frequency of commutation, an approximation of the multivalued $\mathrm{sign}(x)$ function is needed. In particular, this approximation has been successfully tested in simulations and experiments.

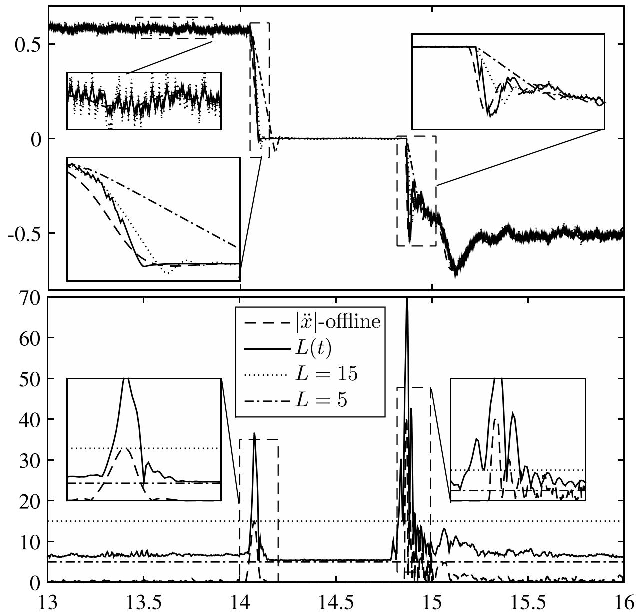

Fig. 6. Top: velocity (in meters per second) versus time (in seconds): STA ($L = 15$), STA ($L = 5$), STAV $L(t)$, and offline. Bottom: gains versus $|\ddot{\hat{x}}|$-offline.

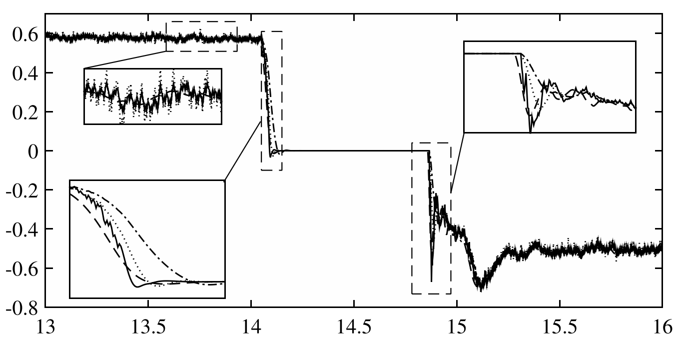

Fig. 7. Velocity (in meters per second) versus time (in seconds): CD ($L = 15$), CD ($L = 5$), TVD $L(t)$, and offline.

we also test the STA algorithm with constant gain. Two values were considered: $L = 5$ and $L = 15$; increasing this value resulted in a high amplitude of chattering. Fig. 6 shows the velocity estimation in the interval of time $(13, 16)$ and the corresponding differentiator gains, constant and time varying. In addition, we computed the estimation of the acceleration $\ddot{x}$ using the presented offline method (see Section IV-C). It is clear that with a constant gain, we cannot compensate for the acceleration in the whole interval. On the other hand, with the variable gain, we can cover the acceleration in the whole region without increasing chattering. The same experiment was realized with the proposed time-varying gain algorithm, confirming a better performance and chattering attenuation with the use of a time-varying gain (see Fig. 7).

G. Comparison of Time-Varying Algorithms

In this section, we present the obtained result with the proposed time-varying algorithms. Fig. 8 shows the velocity estimation in the interval of time $(13, 16)$ s. Fig. 9 shows the velocity estimation in the interval of time $(79, 81.6)$ s and the corresponding time-varying gain $L(t)$. In this interval,

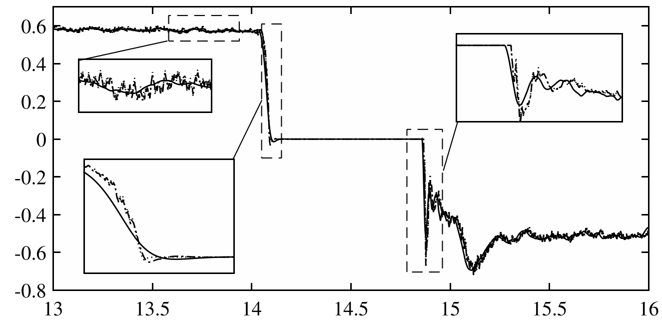

Fig. 8. Velocity (in meters per second) versus time (in seconds): offline, STAV, and TVD.

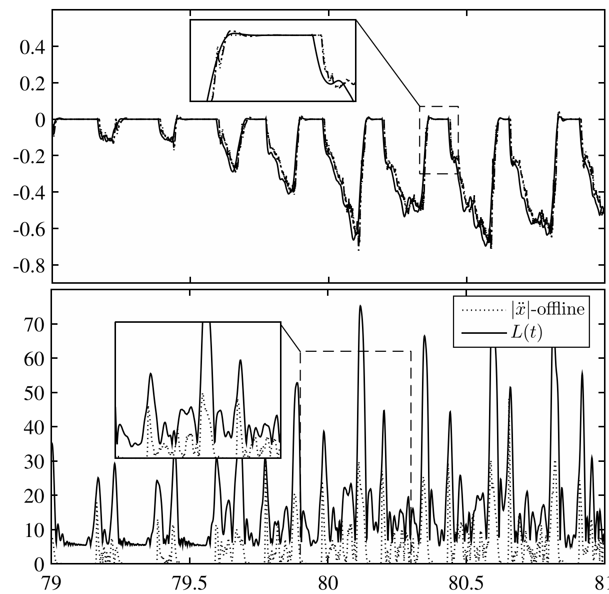

Fig. 9. Top: velocity (in meters per second) versus time (in seconds): offline, STAV $L(t)$, and TVD. Bottom: $L(t)$ versus $|\ddot{\hat{x}}|$-offline.

a sequence of acceleration and deceleration inputs was included in the experiment, producing abrupt changes in velocity. In this case, we cannot use a constant gain, since the gain to select should be $L = 70$, increasing at the same time the chattering effect.

H. Computation of Errors

Considering the offline estimation as the true value of velocity $\dot{x}$, in this section, the second and first norms of the error are computed for the considered intervals of time. For this purpose, we define $e_i = \hat{\dot{x}}_i - \dot{x}$, where $\dot{x}$ is the true velocity (offline estimation) and $\hat{\dot{x}}_i$ is the online estimation for $i = \text{STA, STAV, TVD, and CD}$. Table II shows the normalized error, $||e_i||/||e_{\text{TVD}}||$, during intervals of time that correspond to Figs. 7–9, with $||e_i|| = ((1/t_f - t_0) \int_{t_0}^{t_f} |e_i(\tau)|^2 d\tau)^{1/2}$. In addition, Table III shows the normalized error $||e_i||_1/||e_{\text{TVD}}||_1$, where $||e_i||_1 = \max_{t_0 \le t \le t_f} |e_i|$. In general, the TVD algorithm gives a very good performance and this is the reason to choose e_{TVD} for the normalization. It means that if the value in Table III is below 1, then the corresponding algorithm performs better, then TVD, and vice versa. In the interval of almost constant velocity, $(13, 14)$, the difference in performance is not considerable, except in the case of $L = 15$,

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

TABLE II
$||e_i||_2 \,/\, ||e_{\mathrm{TVD}}||_2$

i	$(13, 14)$ Constant Velocity	$(14, 14.2)$ Abrupt Acceleration	$(79, 81.6)$ Varying Velocity
STA (L=5)	0.8846	5.1255	4.3387
STA (L=15)	1.9810	1.6883	1.9996
STAV	1.0782	1.0277	1.0535
CD (L=5)	0.8333	3.2539	2.7576
CD (L=15)	1.7874	1.5561	1.5546

TABLE III
$||e_i||_\infty \,/\, ||e_{\mathrm{TVD}}||_\infty$

i	$(13, 14)$ Constant Velocity	$(14, 14.2)$ Abrupt Acceleration	$(79, 81.6)$ Varying Velocity
STA (L=5)	0.9084	2.9708	2.6371
STA (L=15)	2.3227	1.2667	1.6295
STAV	1.0906	1.0866	1.0832
CD (L=5)	0.8151	2.2901	1.9651
CD (L=15)	2.0052	1.2743	1.24

where the increase in chattering is evident. The reason for this is that during this interval, the acceleration $\ddot{x}$ decreases to the smallest values, and the amplitude of acceleration can be covered with a relatively small constant gain $L = 5$. Besides, if abrupt changes in acceleration are present, we cannot cover the acceleration amplitude choosing a constant gain. As it can be seen from the columns $(14, 14.2)$ and $(79, 81.6)$ of Tables II and III, the STA algorithm with the constant gain $L = 5$ significantly degrades in performance. Thus, the use of the algorithm with constant gain for the whole operation region results in increase in differentiation errors either for the constant velocity range or for the abrupt acceleration range. However, time-varying algorithms efficiently perform for both profiles of acceleration and provide small errors for the whole operation region.

V. Conclusion

The problem of first-order differentiation under the presence of noise has being studied in this paper. Motivated by applications to mobile hydraulic systems, we look at the situation where certain signals are *a priori* bounded, while a rough estimate for the second derivative can be computed online based on the measurements of pressures. We verify that using a time-varying gain is a better option, instead of using a global constant bound for the whole operation region. The design of differentiators with time-varying gains is presented. Besides, a novel TVD formed by merging the high-gain and second-order sliding mode algorithms is proposed in this paper. A Lyapunov-based analysis has being provided, in order to demonstrate the stability and convergence properties for both algorithms. In addition, the ultimate bounds of the differentiator errors provide a criterion for the enhancement of differentiator parameters. We have tested and validated the proposed scheme on a standard industrial platform of a mobile hydraulic system for forestry, obtaining very good results. The proposed methodologies have shown an increase in performance with respect to the constant gain algorithm, including chattering attenuation. The TVD differentiator allows a good trade-off between the high-gain and the second-order sliding mode, compromising a transient performance and chattering effect. Extensions of this algorithm applied to a more general class of mechanical systems are considered for future work.

Appendix

A. Lyapunov Analysis

Consider the system

$$\dot{\mathbf{x}} = \mathbf{f}(t, \mathbf{x}) \tag{27}$$

where $\mathbf{x}$ and $\mathbf{f}$ are n-D vectors, particularly the vector function $\mathbf{f}$ is piecewise continuous. The precise meaning of a solution of a differential equation (27) with a piecewise continuous right-hand side is understood in the Filippov sense [35], [38]. To analyze asymptotic stability of the origin, it is sufficient to find a continuous positive definite function $V(\cdot)$ such that for any solution $\mathbf{x}(t)$, the function V is monotonically decreasing. The case where the function $V(\cdot)$ is continuously differentiable has been widely studied, see e.g. [41]. Recently, in the surveys [37] and [38], some results concerning the case when $V(\cdot)$ is discontinuous have been pointed out. Next, we are going to summarize some of these results. We refer the reader to [37] and [38] for a more complete study.

B. Derivative Numbers and Monotonicity

In the analysis of discontinuous Lyapunov functions, the theory of contingent derivatives plays an important role [36], [38]. Let $\mathbb{K}$ be a set of all sequences of real numbers converging to zero and let a real-valued function φ be defined on some interval $\mathcal{I}$.

Definition 1: A number $D_{\{h_n\}}\varphi(t) = \lim_{n\to+\infty} (\varphi(t + h_n) - \varphi(t))/h_n$, $\{h_n\} \in \mathbb{K} : t + h_n \in \mathcal{I}$, is called the *derivative number* of the function φ at a point $t \in \mathcal{I}$ if finite or infinite limit exists. The set of all derivative numbers of the function φ at the point $t \in \mathcal{I}$ is called *contingent derivative*

$$D_{\mathbb{K}}\varphi(t) = \bigcup_{\{h_n\}\in\mathbb{K}} \{D_{\{h_n\}}\varphi(t)\} \subseteq \bar{\mathbb{R}}$$

where $\bar{\mathbb{R}} = \mathbb{R} \cup \{-\infty\} \cup \{+\infty\}$.

If a function $\varphi(t)$ is differentiable at a point $t \in \mathcal{I}$, then $D_{\mathbb{K}}\varphi(t) = \{\dot{\varphi}(t)\}$. The contingent derivative helps to prove monotonicity of a nondifferentiable or discontinuous function.

Proposition 5: If a function $\varphi : \mathbb{R} \to \mathbb{R}$ is defined on $\mathcal{I}$ and the inequality $D_{\mathbb{K}}\varphi(t) \leq 0$ holds for all $t \in \mathcal{I}$, then $\varphi(t)$ is a decreasing function on $\mathcal{I}$ and differentiable almost everywhere on $\mathcal{I}$.

Observe that Proposition 5 does not require the continuity of the function φ or the finiteness of its derivative numbers. It gives us a background for the discontinuous Lyapunov function method. The generalized derivatives presented above are closely related to the well-known Dini derivatives.

1) The right-hand upper Dini derivative

$$D^+\varphi(t) = \limsup_{h\to 0^+} \frac{\varphi(t+h) - \varphi(t)}{h}.$$

2) The right-hand lower Dini derivative

$$D_+\varphi(t) = \liminf_{h\to 0^+} \frac{\varphi(t+h)-\varphi(t)}{h}.$$

3) The left-hand upper Dini derivative

$$D^-\varphi(t) = \limsup_{h\to 0^-} \frac{\varphi(t+h)-\varphi(t)}{h}.$$

4) The left-hand lower Dini derivative

$$D_-\varphi(t) = \liminf_{h\to 0^-} \frac{\varphi(t+h)-\varphi(t)}{h}.$$

One can observe that $D_+\varphi(t) \le D^+\varphi(t)$ and $D_-\varphi(t) \le D^-\varphi(t)$. In addition, all Dini derivatives belong to the set $D_{\mathbb{K}}\varphi(t)$ and

$$D_{\mathbb{K}}\varphi(t) \le 0 \iff \begin{cases} D^-\varphi(t) \le 0 \\ D^+\varphi(t) \le 0 \end{cases}$$

$$D_{\mathbb{K}}\varphi(t) \ge 0 \iff \begin{cases} D_-\varphi(t) \ge 0 \\ D_+\varphi(t) \ge 0. \end{cases}$$

It is worth mentioning that all results for contingent derivative can be rewritten in terms of Dini derivatives.

Theorem 1: If $\varphi : \mathbb{R} \to \mathbb{R}$ is a function defined on an interval $\mathcal{I}$, then for almost all $t \in \mathcal{I}$, Dini derivatives of $\varphi(t)$ satisfy one of the following four conditions.

1) $\varphi(t)$ has a finite derivative.
2) $D^+\varphi(t) = D_-\varphi(t)$ is finite, $D^-\varphi(t) = +\infty$, and $D_+\varphi(t) = -\infty$.
3) $D^-\varphi(t) = D_+\varphi(t)$ is finite, $D^+\varphi(t) = +\infty$, and $D_-\varphi(t) = -\infty$.
4) $D^-\varphi(t) = D^+\varphi(t) = +\infty$ and $D_-\varphi(t) = D_+\varphi(t) = -\infty$.

Corollary 1: If $\varphi : \mathbb{R} \to \mathbb{R}$ is a function defined on an interval $\mathcal{I}$, then the equality $D_{\mathbb{K}}\varphi(t) = -\infty$ ($D_{\mathbb{K}}\varphi(t) = +\infty$) may hold only on a subset of measure zero.

C. Generalized Directional Derivatives

If a Lyapunov function is not differentiable, the concept of generalized directional derivatives can be used for the stability analysis. Let $\mathbb{M}(d)$ be a set of all sequences of real vectors converging to $d \in \mathbb{R}^n$, i.e., $\{\upsilon_n\} \in \mathbb{M}(d) \iff \upsilon_n \to d$, $\upsilon_n \in \mathbb{R}^n$. Let a function $V : \mathbb{R}^n \to \mathbb{R}$ be defined on an open nonempty set $\Omega \subseteq \mathbb{R}^n$ and $d \in \mathbb{R}^n$.

Definition 2: A number

$$D_{\{h_n\},\{\upsilon_n\}} V(\mathbf{x}, d) = \limsup_{n\to\infty} \frac{V(\mathbf{x} + h_n \upsilon_n) - V(\mathbf{x})}{h_n}$$
$$\{h_n\} \in \mathbb{K}, \quad \{\upsilon_n\} \in \mathbb{M}(d) : x + h_n \upsilon_n \in \Omega$$

is called the *directional derivative number* of the function $V(\mathbf{x})$ at the point $\mathbf{x} \in \Omega$ on the direction $d \in \mathbb{R}^n$ if finite or infinite limit exists. The set of all directional derivative numbers of the function $V(\mathbf{x})$ at the point $\mathbf{x} \in \Omega$ on the direction $d \in \mathbb{R}^n$ is called *directional contingent derivative*

$$D_{\mathbb{K},\mathbb{M}(d)} V(\mathbf{x}) = \bigcup_{\{h_n\}\in\mathbb{K},\, \{\upsilon_n\}\in\mathbb{M}(d)} \{D_{\{h_n\},\{\upsilon_n\}} V(\mathbf{x}, d)\}.$$

D. Discontinuous Lyapunov Functions

Some basic results concerning discontinuous Lyapunov functions are presented in this section, see [37] and [38] for a complete study.

Definition 3: A function $V : \mathbb{R}^n \to \mathbb{R}$ is said to be proper on an open nonempty set $\Omega \subseteq \mathbb{R}^n : 0 \in \mathrm{int}(\Omega)$ if it satisfies the following conditions.

1) It is defined on Ω and continuous at the origin.
2) There exists a continuous positive definite function $\underline{V}$ such that $\underline{V}(\mathbf{x}) \le V(\mathbf{x})$ for $\mathbf{x} \in \Omega$.

If $\Omega = \mathbb{R}^n$, V is globally proper.

Theorem 2: Let a function $V : \mathbb{R}^n \to \mathbb{R}$ be proper on an open nonempty set $\Omega \subseteq \mathbb{R}^n : 0 \in \mathrm{int}(\Omega)$, satisfying

$$\alpha_0 W(\|\mathbf{x}\|) \le V(\mathbf{x}) \le \alpha_1 W(\|\mathbf{x}\|)$$
$$D_{F(t,x)} V(\mathbf{x}) \le -\beta_0 V(\mathbf{x})$$

then the origin of the system is exponentially stable, where $W(\|x\|) > 0$ and

$$D_{F(t,x)} V(\mathbf{x}) = \bigcup_{d\in F(t,x)} D_{\mathbb{K},\mathbb{M}(d)} V(\mathbf{x}).$$

If $\Omega = \mathbb{R}^n$, the origin is globally exponentially stable.

References

[1] H. K. Khalil and L. Praly, "High-gain observers in nonlinear feedback control," *Int. J. Robust Nonlinear Control*, vol. 24, no. 6, pp. 993–1015, Apr. 2014.

[2] G. Bartolini, A. Pisano, and E. Usai, "First and second derivative estimation by sliding mode technique," *J. Signal Process.*, vol. 4, no. 2, pp. 167–176, 2000.

[3] A. Levant, "Higher-order sliding modes, differentiation and output-feedback control," *Int. J. Control*, vol. 76, nos. 9–10, pp. 924–941, 2003.

[4] A. Pisano and E. Usai, "Globally convergent real-time differentiation via second order sliding modes," *Int. J. Syst. Sci.*, vol. 38, no. 10, pp. 833–844, 2007.

[5] A. Pisano and E. Usai, "Sliding mode control: A survey with applications in math," *Math. Comput. Simul.*, vol. 81, no. 5, pp. 954–979, 2011.

[6] A. Levant, "Robust exact differentiation via sliding mode technique," *Automatica*, vol. 34, no. 3, pp. 379–384, Mar. 1998.

[7] J. Davila, L. Fridman, and A. Levant, "Second-order sliding-mode observer for mechanical systems," *IEEE Trans. Autom. Control*, vol. 50, no. 11, pp. 1785–1789, Nov. 2005.

[8] J. A. Moreno and M. Osorio, "Strict Lyapunov functions for the super-twisting algorithm," *IEEE Trans. Autom. Control*, vol. 57, no. 4, pp. 1035–1040, Apr. 2012.

[9] A. E. Polyakov and A. S. Poznyak, "Method of Lyapunov functions for systems with higher-order sliding modes," *Autom. Remote Control*, vol. 72, no. 5, pp. 944–963, 2011.

[10] N. Orani, A. Pisano, and E. Usai, "On a new sliding-mode differentiation scheme," in *Proc. IEEE Int. Conf. Ind. Technol.*, Dec. 2006, pp. 2652–2657.

[11] D. V. Efimov and L. Fridman, "A hybrid robust non-homogeneous finite-time differentiator," *IEEE Trans. Autom. Control*, vol. 56, no. 5, pp. 1213–1219, May 2011.

[12] V. Utkin, "On convergence time and disturbance rejection of super-twisting control," *IEEE Trans. Autom. Control*, vol. 58, no. 8, pp. 2013–2017, Aug. 2013.

[13] V. I. Utkin and A. S. Poznyak, "Adaptive sliding mode control with application to super-twist algorithm: Equivalent control method," *Automatica*, vol. 49, no. 1, pp. 39–47, 2013.

[14] Y. Shtessel, M. Taleb, and F. Plestan, "A novel adaptive-gain *supertwisting* sliding mode controller: Methodology and application," *Automatica*, vol. 48, no. 5, pp. 759–769, 2012.

[15] T. Gonzalez, J. A. Moreno, and L. Fridman, "Variable gain super-twisting sliding mode control," *IEEE Trans. Autom. Control*, vol. 57, no. 8, pp. 2100–2105, Aug. 2013.

[16] E. Cruz-Zavala, J. A. Moreno, and L. M. Fridman, "Uniform robust exact differentiator," *IEEE Trans. Autom. Control*, vol. 56, no. 11, pp. 2727–2733, Nov. 2011.
[17] M. T. Angulo, J. A. Moreno, and L. Fridman, "Robust exact uniformly convergent arbitrary order differentiator," *Automatica*, vol. 49, no. 8, pp. 2489–2495, 2013.
[18] H. Alwi and C. Edwards, "An adaptive sliding mode differentiator for actuator oscillatory failure case reconstruction," *Automatica*, vol. 49, no. 2, pp. 642–651, 2013.
[19] L. Sidhom, X. Brun, M. Smaoui, E. Bideaux, and D. Thomasset, "Dynamic gains differentiator for hydraulic system control," *J. Dyn. Sys., Meas., Control*, vol. 137, no. 4, pp. 041017-1–041017-13, 2015.
[20] C. Vázquez, S. Aranovskiy, and L. Freidovich, "Time-varying gain second order sliding mode differentiator," in *Proc. 19th IFAC World Congr.*, Cape Town, South Africa, 2014, pp. 1374–1379.
[21] A. Levant and M. Livne, "Exact differentiation of signals with unbounded higher derivatives," *IEEE Trans. Autom. Control*, vol. 57, no. 4, pp. 1076–1080, Apr. 2012.
[22] H. E. Merritt, *Hydraulic Control Systems*. New York, NY, USA: Wiley, 1967.
[23] J. Komsta, N. van Oijen, and P. Antoszkiewicz, "Integral sliding mode compensator for load pressure control of die-cushion cylinder drive," *Control Eng. Pract.*, vol. 21, no. 5, pp. 708–718, 2013.
[24] C. Guan and S. Pan, "Adaptive sliding mode control of electro-hydraulic system with nonlinear unknown parameters," *Control Eng. Pract.*, vol. 16, no. 11, pp. 1275–1284, 2008.
[25] K. K. Ahn, D. N. C. Nam, and M. Jin, "Adaptive backstepping control of an electrohydraulic actuator," *IEEE/ASME Trans. Mechatronics*, vol. 19, no. 3, pp. 987–995, Jun. 2013.
[26] S. Aranovskiy, "Modeling and identification of spool dynamics in an industrial electro-hydraulic valve," in *Proc. 21st Medit. Conf. Control Autom.*, Jun. 2013, pp. 82–87.
[27] S. Aranovskiy, A. Losenkov, and C. Vázquez, "Position control of an industrial hydraulic system with a pressure compensator," in *Proc. 22nd Medit. Conf. Control Autom.*, Palermo, Italy, Jun. 2014, pp. 1329–1334.
[28] A. Bonchis, P. I. Corke, and D. C. Rye, "Experimental evaluation of position control methods for hydraulic systems," *IEEE Trans. Control Syst. Technol.*, vol. 10, no. 6, pp. 876–882, Nov. 2002.
[29] W. Kim, D. Shin, D. Won, and C. C. Chung, "Disturbance-observer-based position tracking controller in the presence of biased sinusoidal disturbance for electrohydraulic actuators," *IEEE Trans. Control Syst. Technol.*, vol. 21, no. 6, pp. 2290–2298, Nov. 2013.
[30] C. Vázquez, S. Aranovskiy, and L. Freidovich, "Sliding mode control of a forestry-standard mobile hydraulic system," in *Proc. 13th Int. Workshop Variable Struct. Syst.*, Nantes, France, Jun./Jul. 2014, pp. 1–6.
[31] D. Won, W. Kim, D. Shin, and C. C. Chung, "High-gain disturbance observer-based backstepping control with output tracking error constraint for electro-hydraulic systems," *IEEE Trans. Control Syst. Technol.*, vol. 23, no. 2, pp. 787–795, Mar. 2015.
[32] L. K. Vasiljevic and H. K. Khalil, "Error bounds in differentiation of noisy signals by high-gain observers," *Syst. Control Lett.*, vol. 57, no. 10, pp. 856–862, 2008.
[33] A. N. Kolmogorov, "On inequalities between upper bounds of consecutive derivatives of an arbitrary function defined on an infinite interval," *Amer. Math. Soc. Transl.*, vol. 9, no. 2, pp. 233–242, 1962.
[34] V. Utkin, J. Guldner, and J. Shi, *Sliding Mode Control in Electro-Mechanical Systems*, 2nd ed. London, U.K.: Taylor & Francis, 2009.
[35] A. F. Filippov, *Differential Equations With Discontinuous Righthand Sides*. Norwell, MA, USA: Kluwer, 1988.
[36] I. P. Natanson, *Theory of Functions of a Real Variable*. New York, NY, USA: Frederick Unger, 1955.
[37] A. Polyakov, "Discontinuous Lyapunov functions for nonasymptotic stability analysis," in *Proc. 19th IFAC World Congr.*, Cape Town, South Africa, 2014, pp. 5455–5460.
[38] A. Polyakov and L. Fridman, "Stability notions and Lyapunov functions for sliding mode control systems," *J. Franklin Inst.*, vol. 351, no. 4, pp. 1831–1865, 2014.
[39] E. Papadopoulos, B. Mu, and R. Frenette, "On modeling, identification, and control of a heavy-duty electrohydraulic harvester manipulator," *IEEE/ASME Trans. Mechatronics*, vol. 8, no. 2, pp. 178–187, Jun. 2003.
[40] L. Biagiotti and C. Melchiorri, *Trajectory Planning for Automatic Machines and Robots*. Berlin, Germany: Springer-Verlag, 2008.
[41] H. K. Khalil, *Nonlinear Systems*, 3rd ed. Englewood Cliffs, NJ, USA: Prentice-Hall, 2002.

Carlos Vázquez (M'12) received the B.S. degree in electronics engineering from the Celaya Institute of Technology, Celaya, Mexico, in 2003, and the M.S. and Ph.D. degrees from the Automatic Control Department, Center for Research and Advanced Studies, National Polytechnic Institute, Mexico City, Mexico, in 2006 and 2010, respectively.

He was a Post-Doctoral Fellow with the Department of Control Engineering and Robotics, Division of Electrical Engineering, Engineering Faculty, Universidad Nacional Autónoma de México, Ciudad Universitaria, Coyoacán, Mexico, from 2011 to 2013, and the Department of Applied Physics and Electronics, Umeå University, Umeå, Sweden, from 2013 to 2015. He joined Ålö AB, Umeå, as a Research and Development Engineer in the control systems team in 2015.

Stanislav Aranovskiy (M'10) received the Engineering and Ph.D. degrees in systems analysis and control from ITMO University, Saint Petersburg, Russia, in 2006 and 2009, respectively.

He was a Researcher with ITMO University until 2012. From 2012 to 2013, he was with the Department of Applied Physics and Electronics, Umeå University, Umeå, Sweden, as a Post-Doctoral Fellow. In 2014, he returned to ITMO University, where he was a Docent. His current research interests include observers design for nonlinear systems, identification and rejection of disturbances, and adaptive control.

Leonid B. Freidovich (M'06–SM'11) received the M.Sc. degree in mechanics and engineering and the Kandidat of Physical and Mathematical Sciences degree from Saint-Petersburg State Polytechnical University, Saint Petersburg, Russia, in 1996 and 1999, respectively, and the Ph.D. degree in mathematics from Michigan State University, East Lansing, MI, USA, in 2005.

He has been with Umeå University, Umeå, Sweden, since 2005, where he is currently an Associate Professor with the Department of Applied Physics and Electronics, and has been appointed Docent in Control Systems. His current research interests include analysis and control of nonlinear systems with applications in robotics and automation.

Leonid M. Fridman (M'98) received the M.S. degree in mathematics from Kuibyshev (Samara) State University, Samara, Russia, in 1976, the Ph.D. degree in applied mathematics from the Institute of Control Science, Moscow, Russia, in 1988, and the D.Sc. degree in control science from the Moscow State University of Mathematics and Electronics, Moscow, in 1998.

He was with the Department of Mathematics, Samara State Architecture and Civil Engineering University, Samara, from 1976 to 1999. From 2000 to 2002, he was with the Department of Postgraduate Study and Investigations, Chihuahua Institute of Technology, Chihuahua, Mexico. In 2002, he joined the Department of Control Engineering and Robotics, Division of Electrical Engineering, Engineering Faculty, Universidad Nacional Autónoma de México, Ciudad Universitaria, Coyoacán, Mexico. He was an Invited Professor in 20 universities and research laboratories of Argentina, Australia, Austria, France, China, Germany, Italy, Israel, and Spain. He has authored and edited eight books and 15 special issues devoted to the sliding mode control. His current research interests include variable structure systems.

Prof. Fridman was a winner of the Scopus prize for the best cited Mexican Scientists in mathematics and engineering in 2010. He is currently a Chair of the Technical Committee on Variable Structure Systems and Sliding Mode Control of the IEEE Control Systems Society, and an Associate Editor of the *Journal of The Franklin Institute* and *Nonlinear Analysis: Hybrid Systems*.