Manipulators Fault Diagnosis via Higher Order Sliding Mode Observers

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Abstract—A diagnostic scheme for actuator and sensor faults which can occur on a robot manipulator using a model-based Fault Diagnosis (FD) technique is addressed. With the proposed FD scheme it is possible to detect a fault, which can occur on a specific component of the system. To detect actuator faults, higher order sliding mode Unknown Input Observers (UIO) are proposed to provide the necessary analytical redundancy. The detection of sensor faults instead is made by relying on a Generalized Observers Scheme (GOS). The observers input laws are designed according to two well-known second order sliding mode (SOSM) approaches: the so-called Super-Twisting and the Sub-Optimal one. Both typologies of input laws allow to perform a satisfactory fault diagnosis. The peculiarities of each input law of the observers are discussed. To make possible fault isolation, it is required that a single fault acts only on one component of the system at a time. If one knows that faults are occurred only on actuators, then it is possible to isolate multiple simultaneous faults on actuators. The proposed approach is verified in simulation and experimentally on a COMAU SMART3-S2 robot manipulator.

I. INTRODUCTION

Industrial plants and consumer devices often have crucial applications in every-day life. The increasing possibility of reducing the efforts needed by the people to manage and to monitor them is an important challenge. To comply with this challenge, the controllers of these plants should be able to operate also in critical conditions, i.e. when the behavior of some components of the systems is significantly different from the expected behavior. Faults can be caused by particular environmental conditions and by plant conditions itself. They can occur in an unpredictable way on a particular component of the system. Some kind of faults can cause critical injuries to the plant operators and to the plant itself. Then, it is fundamental to include in the controller a block dedicated to diagnose the system. This block should have the capability to make a prompt detection of the fault events [1], [5], [6].

The presence of a fault can be modeled as an unexpected change in the dynamics of the system, in the system parameters or as the presence of unknown signals in the plant. In a robot manipulator, a fault can occur on a specific actuator, on a specific sensor or on a mechanical component of the system. The occurrence of actuator and sensor faults are more frequent, because of the presence of electrical devices, which may be subject to many possible critical situations.

Diagnostic devices are introduced to generate online diagnostic signals which are useful to detect and isolate the fault. The diagnostic signals useful to detect the presence of a fault are usually called residual signals. These signals are obtained from the applied system inputs and the measurements. Residual generators are typically based on observers (see, for instance, [2], [7]–[10]). However, noise and uncertainties can reduce the performances of the observers. Particular techniques are adopted in order to overcome this drawback, such as the use of linear filters [9], generalized momenta, see [11], or Kalman filters [12]. These techniques, in the presence of uncertainties typical of practical applications, cannot guarantee an exact convergence of the observer state to the system state. To reduce this problem, sliding mode based techniques are also frequently adopted to accomplish the state observation [13], [14] because of their design simplicity and robustness features. Usually, the Fault Diagnosis (FD) can be dealt with by combining multiple sliding mode observers as discussed in [9], [15]–[18]. Stochastic based observers can be considered to obtain the residual signals (see [19]). These have proven capabilities to compensate for the effects of noise and are particularly robust for fault diagnosis in a networked environment. Moreover, stochastic observers succeed more accurate estimation and exhibit smoother variations of the estimation error.

In this paper, a fault diagnosis scheme to deal with actuator faults or with sensor faults in robot manipulators is considered. It is based on Unknown Input Observers (UIO) (see [3], [13], [20]–[24]) to detect actuator faults, and on a Generalized Observers Scheme (GOS) to detect sensor faults. While the UIO approach is specifically designed to obtain a suitable estimation of an unknown signal present on the inputs of the system, which can be produced by a fault, the detection of unknown signals which affect the outputs of the system is more difficult because of lack of observability properties typical of the robotic systems, see [18]. The adopted GOS allows to improve the observability properties.

The objective of this work is to study the performance in terms of robustness and diagnostic capabilities of two possible sliding mode input laws for the observers. In particular, two second order sliding mode laws, the Super-Twisting law [21]
and of the Sub-Optimal one [25], are considered. The reason why a second order sliding mode input law turns out to be an effective choice to solve the problem under concern is due to the necessity of coming up with continuous observer input laws, so that such laws can be used as signals upon which the diagnostic procedure is directly based. The diagnostic scheme proposed in this paper proves to be able to detect non simultaneous sensor and actuator faults, and, in some cases, also provides good isolation and identification capabilities independently from the specific second order sliding mode input law which is adopted.

The analysis of both approaches confirms the good features of both algorithms, as well as their suitability to be applied in the fault diagnosis context. Moreover, it also puts into evidence some peculiarities of the two, which can help the user to select between them depending on the specific needs of the considered case. Experimental results are presented in the paper for an industrial manipulator.

II. THE CONSIDERED FAULT SCENARIOS

In this paper, the case of faults occurring on the inputs or on the outputs of a robot manipulator is considered. In the first case, the real torque applied by the actuators is unknown. That is, \( \tau \in \mathbb{R}^n \) being the nominal torque calculated by the robot controller, while \( \Delta \tau \in \mathbb{R}^n \) being the input fault, the actual torque vector which is the input of the robotic system, can be expressed as \( \tau(t) + \Delta \tau(t) \) (see Fig. 1). In case of sensor faults, the control system cannot determine the exact angular displacements of the joints. Let \( q \in \mathbb{R}^n \) be the true but unknown output (i.e. the joints displacements), while \( \Delta q \) is the vector of the fault signals acting on it. Then, \( \hat{q} \in \mathbb{R}^n \) represents the value that the control system receives, i.e., \( \hat{q}(t) = q(t) + \Delta q(t) \) (see Fig. 1).

III. THE MANIPULATOR MODEL

In absence of faults, the dynamics of a \( n \)-joints robot manipulator can be written in the joint space, by using the Lagrangian approach, as

\[
\tau = B(q) \ddot{q} + C(q, \dot{q}) \dot{q} + g(q) + F_e \dot{q} = B(q) \ddot{q} + n(q, \dot{q}) \tag{1}
\]

where \( q \in \mathbb{R}^n \) is the generalized coordinates vector, \( B(q) \in \mathbb{R}^{n \times n} \) is the inertia matrix, \( C(q, \dot{q}) \in \mathbb{R}^{n \times n} \) represents centripetal and Coriolis torques, \( F_e \in \mathbb{R}^{n \times n} \) is the viscous friction diagonal matrix, and \( g(q) \in \mathbb{R}^n \) is the vector of gravitational torques. In this paper, it is assumed that the term \( n(q, \dot{q}) \) can be identified, while the term \( B(q) \) is regarded as known.

Now, introducing the variables \( \chi_1(t) = q(t), \chi_2(t) = \dot{q}(t) \), when faults affect either the actuators or the sensors, model (1) can be rewritten in the state space representation as

\[
\begin{align*}
\dot{\chi}_1(t) &= \chi_2(t) \\
\dot{\chi}_2(t) &= f(\tau(t) + \Delta \tau(t), \chi_1(t), \chi_2(t)) \\
\hat{q}(t) &= \chi_1(t) + \Delta \tau(t).
\end{align*}
\]

where \( f(\cdot) \) is obtained after an identification procedure such as the one proposed in [4], and it is given by

\[
\hat{f}(\tau + \Delta \tau, \chi_1, \chi_2) = \tau^{-1}(\chi_1) (\tau + \Delta \tau - \check{n}(\chi_1, \chi_2)) \tag{3}
\]

and \( \check{n}(q, \dot{q}) \) is the known part of the model such that \( n(\chi_1, \chi_2) = n(\chi_1, \chi_2) - \check{n}(\chi_1, \chi_2) \). The term \( \eta(t) \) is uncertain but can be assumed to be bounded, since in real applications the terms \( q \) and \( \dot{q} \) are bounded, see [27].

IV. ACTUATORS FAULT DIAGNOSIS STRATEGY

By relying on the so-called Unknown Input Observer (UIO) approach [13], efficient estimators of the input torques can be designed [6], [28]. In this paper, we propose to detect the actuator faults by means of UIOs of sliding mode type as shown in the next subsection.

A. Observer Design

Let us consider the observer

\[
\begin{align*}
\dot{\hat{x}}_1(t) &= \hat{x}_2(t) + z_1(t) \\
\dot{\hat{x}}_2(t) &= \hat{f}(\tau(t), \chi_1(t), \hat{x}_2(t)) + z_2(t)
\end{align*}
\]

where \( \hat{x}_1(t) \in \mathbb{R}^n, \hat{x}_2(t) \in \mathbb{R}^n \) are the observer states, and \( z(t) = [z_1(t), z_2(t)]^T \) is an auxiliary input signal, which is designed relying on an sliding mode approach. This signal is introduced in order to guarantee the convergence of the observer states to the actual state of the system.

B. Dynamics of the Observer Error

The proposed fault diagnostic scheme requires to steer to zero the signal \( e(t) = [e_1(t), e_2(T)]^T \in \mathbb{R}^{2n} \), the components of which are given by

\[
\begin{align*}
e_1(t) &= \chi_1(t) - \hat{\chi}_1(t) \\
e_2(t) &= \chi_2(t) - \hat{\chi}_2(t).
\end{align*}
\]

By steering to zero these quantities, it is possible to guarantee that the observer (4) gives a good estimation of the unknown input. The dynamics of the error variable \( e(t) \) is represented by a second order dynamical system,

\[
\begin{align*}
\dot{e}_1(t) &= e_2(t) - z_1(t) \\
\dot{e}_2(t) &= \tau^{-1}(\chi_1(t)) (\Delta \tau(t) - \eta(\chi_1, \chi_2, \hat{x}_2)) - z_2(t).
\end{align*}
\]

Now, two different Second Order Sliding Mode approaches are exploited to design the multi-input-multi-state UIO input law. The first approach is the so-called Super-Twisting algorithm [21], while the second is the so-called Sub-Optimal algorithm.
C. Super-Twisting based Observer

The design of the observer input laws which are the components of $z(t) = [z_1(t), z_2(t)]^T$ using a Super-Twisting based approach (see [21]) is given by

$$\begin{align*}
z_1(t) &= \lambda \sqrt{\sigma(t)} \text{sign}(\sigma(t)) \\
z_2(t) &= \text{assign}(\sigma(t)) \tag{7}
\end{align*}$$

where $\sigma(t) = e_1(t) = \chi_1(t) - \hat{\chi}_1(t)$ and on the basis of [21], the selection of parameters $\alpha$ and $\lambda$ and can be made taking into account the constraints

$$\alpha > f^+, \lambda > \sqrt{\frac{2}{\alpha - f^+} (1 + \nu)} \tag{8}$$

where $f^+$ is an upper bound of the term $f(x_1, x_2)$, which in our case has been experimentally determined and $\nu$ is a constant, $0 < \nu < 1$.

Note that the term $z_2(t)$ is a discontinuous signal and, by virtue of the filtering action considered in [21], the second equation of the system (6) can be rewritten as

$$z_{2eq}(t) = B^{-1}(\chi_1(t))(\Delta T(t) - \eta(t)) \tag{9}$$

where $z_{2eq}(t)$ is the equivalent input signal corresponding to the discontinuous signal $z_2(t)$, see [29]. Thus, theoretically, the equivalent input signal is the result of an infinite switching frequency of the discontinuous term $\text{assign}(\sigma(t))$. In fact, the implementation of the observer produces high switching frequency making necessary the application of a filter to obtain useful information from signal $z_2(t)$. The filter has to eliminate the high frequency components of such a signal. In [30] and [31], were shown that is possible to assume that the equivalent input law (9) is equivalent to a filter output.

D. Sub-Optimal Algorithm based Observer

The observer input laws of Sub-Optimal type are given by

$$\begin{align*}
z_1(t) &= 0, \quad z_2(0) = 0 \\
z_2(t) &= \delta W \text{sign} \left\{ \sigma''(t) - 0.5 \sigma_M^2 \right\} \tag{10}
\end{align*}$$

where $\sigma''(t) = e_2(t) + \beta e_1(t)$ is the sliding variable and, in this case, $\sigma_M^2$ represents the last extremal value of the sliding variable $\sigma(t)$, and $\beta > 0$. The last extremal value of the sliding variable $\sigma''(t)$ can be found following two possible approaches: the peak detection algorithm proposed in [25], [32] or considering the output of a second order robust differentiator algorithm, see [33].

It can be proved that a suitable choice of $\delta W$ exists such that the Sub-Optimal input laws guarantee the exponential stability of the tracking error of this observer. In (10), on the basis of [25], the selection of parameters $W$ and $\delta$ can be made taking into account the constraints

$$W > \max \left( \frac{f^+}{\alpha^*}, \frac{4f^+}{3 - \alpha^*} \right), \quad f^+ > |\eta|, \quad \alpha^* > 0$$
$$\delta_i = \begin{cases} \alpha^* & \text{if } |\sigma''(t) - 0.5 \sigma_M^2| |\sigma''(t) - \sigma''(t)| > 0 \\ 1 & \text{if } |\sigma''(t) - 0.5 \sigma_M^2| |\sigma''(t) - \sigma''(t)| \leq 0 \end{cases} \tag{11}$$

E. Residual Generation for Actuator Faults

The residual signal considered for fault diagnosis is obtained in both cases from the input law $z_2(t)$.

As previously stated, to obtain suitable residual signals from the discontinuous signals of the observers, it is necessary to apply a filter. Let us consider the following 5th order low-pass filter ($s$ is the Laplace operator)

$$F(s) = \frac{b}{1 - as^{-1} - as^{-2} - as^{-3} - as^{-4} - as^{-5}}. \tag{12}$$

where $a = 0.1961$ and $b = 15a$ have been chosen after an experimental tuning procedure.

The residual signal useful to detect actuator faults is given by

$$r_i(t) = \begin{cases} 0 & \text{if } |\ell(t) + B(q(t))z_2(t)| < T_i \\ 1 & \text{if } |\ell(t) + B(q(t))z_2(t)| > T_i \end{cases} \tag{13}$$

where $\ell(t)$ indicates the impulse response of $F(s)$, $\ast$ indicates the convolution product, and $T_i$ are suitable thresholds chosen on the basis of the amplitude of the noise which is present on the system.

F. Identification of the Actuator Faults Signals

The input signal $z_2(t)$, independently of the type of the input law adopted, between the two proposals previously described, is useful also to give an estimation of the shape of the fault signal $\Delta \tau$. That is, the estimation $\hat{\Delta \tau}$ of the input fault $\Delta \tau$ is given by

$$\hat{\Delta \tau} = \ell(t) * B(q(t))z_2(t) \tag{14}$$

and $z_2(t)$ can be obtained from both (7) or (10). The term $\eta(t)$ is just neglected in the present work and no further considerations on its bounds are proposed for robustness purposes. Additionally, the choice of the thresholds is based on the experimental determination of upper bounds on noise and unmodeled effects. Proposals of choices of the thresholds, for instance based on optimality criteria, can be found in [34].

Theorem IV.1 (Convergence of the $\hat{\Delta \tau}$ to $\Delta \tau$ by using the Super-Twisting input laws). Using the input laws (7) in the observer (4), a choice of the terms $\alpha$ and $\lambda$ exists according to (8) such that the condition $|\epsilon_1(t), e_2(t)|^T = 0, 0$ being the null vector in $\mathbb{R}^{2n}$, is reached in finite time. Then, in absence of noise $\epsilon(t)$, the signal $\hat{\Delta \tau}$ converges to $\Delta \tau$ exponentially.

The condition $|\epsilon_1(t), e_2(t)|^T = 0$ is achieved in finite time as it was developed in [21] (see Theorem 1).

A similar result can be established for the Sub-Optimal input law (10).

Theorem IV.2 (Convergence of the $\hat{\Delta \tau}$ to $\Delta \tau$ by using the Sub-Optimal input laws). Using the input laws (10) in the observer (4), a choice of the terms $W_{1,i,\delta}$ and $\alpha^*$ exists according to (11) such that the observer error state vector $|\epsilon_1(t), e_2(t)|^T$ reaches the origin exponentially. Then, in absence of noise $\epsilon(t)$, the signal $\Delta \tau$ converges to $\Delta \tau$ exponentially.

The proof that the condition $|\epsilon_1(t), e_2(t)|^T = 0$ is achieved exponentially was developed in [25] and [27] (see Theorems 1 and 2, respectively).
V. SENSORS FAULT DIAGNOSIS STRATEGY

To perform the detection of sensor faults, $n$ observers are used, one for each sensor (see Fig. 2). This strategy, called Generalized Observer Scheme (GOS) [8], and the proposed particular implementation are described in this section.

In the sequel, the following notation is considered for the vectors used in the GOS observers: $\hat{z}^{(i)}_1 \in \mathbb{R}^n$ is the vector of the estimate of the $q$ vector made by the $i$th observer, $e^{(i)} \in \mathbb{R}^n$ contains the corresponding observation errors, while $p^{(i)} \in \mathbb{R}^n$ is the input law of observer $i$. Moreover, the components of $\hat{z}^{(i)}_1$ are $\hat{x}^{(i)}_1 = [\hat{x}^{(i)}_{1,1}, \hat{x}^{(i)}_{1,2}, \ldots, \hat{x}^{(i)}_{1,n}]^T$, $i = 1, \ldots, n$. Now, the $i$th GOS observer in our proposal is defined as

$$
\begin{aligned}
\dot{\hat{x}}^{(i)}_1 &= \hat{x}^{(i)}_2 + z^{(i)}_1(t) \\
\dot{z}^{(i)}_2 &= f(\hat{x}^{(i)}_1, \hat{x}^{(i)}_2, \tau) + p^{(i)}(t)
\end{aligned}
$$

(15)

In contrast to the actuator FD, for which a single observer is used, for sensor FD, $n$ sliding mode observers are designed.

Then, also in this case, we design the input laws $z^{(i)}_1(t)$ and $p^{(i)}(t)$ of each observer in (15) with the Sub-Optimal and Super-Twisting algorithms.Posing the $i$th component of $z^{(i)}_1(t)$ and $p^{(i)}(t)$ equal to zero, i.e., for the Sub-Optimal Algorithm

$$
\rho^{(i)}_j(t) = \sigma_j \\n\rho^{(i)}_j(t) = 0, \quad j = i, \quad z^{(i)}_1(t) = 0
$$

(16)

and for the Super-Twisting Algorithm

$$
\begin{aligned}
\dot{z}^{(i)}_j(t) &= \lambda_j \sqrt{\sigma_j^{(i)}(t)} \text{sign}(\sigma_j^{(i)}(t)), \quad j \neq i \\
\rho^{(i)}_j(t) &= \alpha_j \text{sign}(\sigma_j^{(i)}(t)), \quad j \neq i \\
\rho^{(i)}_j(t) &= 0, \quad z^{(i)}_j(t) = 0, \quad j = i \\
\sigma^{(i)}(t) &= \varepsilon^{(i)} + \beta \varepsilon^{(i)}
\end{aligned}
$$

(17)

with $j$ is the index of the component of the state vector coinciding with the sensor number, while $\lambda_j$ and $\alpha_j$, $\sigma_j^{(i)}$ and $\beta > 0$ are analogous to that defined for the actuators.

Note that, with this input law, the observer (15) has the $i$th component in open loop. This can imply stability problems also in absence of faults, if the system is not open loop stable by itself. To circumvent stability problems while avoiding a significant reduction of the detection performances, in the experimental tests, a local small gain proportional-derivative feedback is closed to generate the $i$th input law component of the $i$th observer (16). That is,

$$
\rho^{(i)}_1(t) = K_p \left( q_i + \Delta q_i - \hat{x}^{(i)}_1 \right) + K_d \left( -\hat{x}^{(i)}_2 \right)
$$

(18)

for all $i$, with $K_p > 0$ and $K_d > 0$ small constants.

More precisely, if $f(t)$ can be assumed to be a quite accurate estimate of $f(\cdot)$, a fault can be detected considering Table I. If, in contrast, $f(t)$ differs from $f(\cdot)$, thresholds need to be introduced. The entries of Table I expressed as $\rho^{(i)}_1 \neq 0$ can be replaced by the condition

$$
\left\lfloor \text{sign} \left\{ \rho^{(i)}_1 \right\} \right\rfloor < T_k, \quad \forall k = 1, \ldots, n
$$

(19)

while the entries of Table I expressed as $\rho^{(i)}_1 = 0$ become

$$
\left\lfloor \text{sign} \left\{ \rho^{(i)}_1 \right\} \right\rfloor > T_k
$$

(20)

where $\text{sign}\{\rho^{(i)}_1\}$ is the vector containing the sign of each component of $\rho^{(i)}_1$, the symbol $\circ$ denotes the Schur product, $\left\lfloor \cdot \right\rfloor_k$ denotes the $k$th component of a vector, and $T_k$ is a positive real number representing the selected threshold.

A. Fault Diagnosis Logic: Residual Generation and Fault Isolation

Fig. 1 shows the complete diagnostic scheme for robot manipulators. The residual vector $r_i$ is given by

$$
r_{r_i} = \begin{cases} 0 & \text{if } |F_i(t) + \rho^{(i)}_1| < T_i^* \\ 1 & \text{if } |F_i(t) + \rho^{(i)}_1| > T_i^* \end{cases} \quad \forall i
$$

(21)

while the residual vector $r_q$ is obtained by filtering the $\rho^{(i)}_{1 \neq i}(t)$ signals through the filter (12) and comparing these signals with their thresholds $T_k$, according to Table I.

<table>
<thead>
<tr>
<th>Fault</th>
<th>$r_{r_{r1}}$</th>
<th>$r_{r_{r2}}$</th>
<th>$r_{r_{rn}}$</th>
<th>$r_{r_{ql}}$</th>
<th>$r_{r_{q2}}$</th>
<th>$r_{r_{qn}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Act. 1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Act. 2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>p</td>
<td>p</td>
<td>p</td>
</tr>
<tr>
<td>Act. n</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>p</td>
<td>p</td>
<td>p</td>
</tr>
<tr>
<td>Sens. 1</td>
<td>p</td>
<td>p</td>
<td>p</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Sens. 2</td>
<td>p</td>
<td>p</td>
<td>p</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Sens. n</td>
<td>p</td>
<td>p</td>
<td>p</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

TABLE II

| Fault signature table. |

The isolation of a fault can be performed by comparing the binary detection vector $[r_{r}, r_{q}]$ with the fault signature Table II.
VI. A CASE STUDY

A. The Considered Manipulator

The fault diagnosis technique described in this paper has been experimentally verified on a COMAU SMART3-S2 anthropomorphic rigid robot manipulator which is a classical example of industrial manipulator (see Fig. 3). It consists of six links and six rotational joints driven by brushless electric motors. During the experiments the robot has been constrained to move on a vertical plane. Then, it is possible to consider the robot as a three link-three joint, in the sequel numbered as \{1, 2, 3\}, planar manipulator (see Fig. 3). Yet, the method proposed in this paper holds for \( n \)-joints robots even of spatial type.

The controller has a sampling time of 0.001s, a 12 bit D/A and a 16 bit A/D converters. The joints positions are acquired by resolvers fastened on the three motors, holding mechanical reducers with ratio \{207, 60, 37\} respectively, while the maximum torques are \{1825, 528, 71\}[Nm].

B. Experimental Tests

The faults are introduced in the control system by adding a fault signal to the 3-dimensional control variable, or to the 3-dimensional measured displacements for the sensor faults.

The parameters for the Sub-Optimal algorithm, chosen according to the parameters selection rules indicated in [25], are \( U_M = [362, 607, 10286]^T \), with \( \beta = 10 \) while the parameter choice for the Super-Twisting is given by \( \lambda = [7, 4, 17]^T \) and \( \alpha = [1.6, 2.0, 2.4]^T \) according to equation (7) in [21].

C. Experimental Fault Diagnosis on Actuators

The case of abrupt faults on the actuators of each joint are considered, that is a \(-50\)[Nm] fault signal acting on the first actuator, a \(-20\)[Nm] fault signal acting on the second actuator, and a \(-10\)[Nm] fault signal acting on the third actuator are considered. Note that these faults signals are approximately the 20% of the maximum torque allowed by the corresponding actuator. In Fig. 4 for the Super-Twisting UIOs and in Fig. 5 for the Sub-Optimal UIOs, the fault signals are correctly detected, isolated, and identified. The Super-Twisting approach, as for the fault occurred on the first and second actuators, provides better performances, while as for the fault occurred on the third actuator, the Sub-Optimal approach shows a superior capability to avoid false alarms. Note that, theoretically, Sub-Optimal and Super-Twisting differentiators have the same performance and this is confirmed by our experiments. The difference stands in the fact that for realizing the differentiation process using the sub-optimal algorithm a peak detector is required limiting the maximum value of the incoming signal. Super-twisting differentiator requires only the input signal. In turn, this can require the same filtering of the input. As a consequence, we have seen that one can use lower thresholds for the Super-Twisting making the detection more precise, and higher for the Sub-Optimal, reducing the false alarm rate.

D. Experimental Fault Diagnosis on Sensors

In the case of faults occurring on the sensors the fault signature given in Table I is considered. The plots of Figs. 6 and 7 show the two components different from zero of the input laws of the three GOS observers. In Fig. 6, are shown the results obtained by applying the Sub-Optimal approach, while Fig. 7 reports the results obtained by applying the Super-Twisting approach. The fault has occurred on sensor 2. Both approaches can detect and isolate the fault but they have different performances in terms of possibility of making a reliable detection: the Sub-Optimal strategy has produced a more clear response than that obtained with the Super-Twisting strategy.

<table>
<thead>
<tr>
<th>Signal</th>
<th>Sub-Optimal</th>
<th>Super-Twisting</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{\Delta}r_1(t) )</td>
<td>55-18=37[Nm]</td>
<td>58-20=38[Nm]</td>
</tr>
<tr>
<td>( \tilde{\Delta}r_2(t) )</td>
<td>25-7=18[Nm]</td>
<td>23-11=12[Nm]</td>
</tr>
<tr>
<td>( \tilde{\Delta}r_3(t) )</td>
<td>11-2=9[Nm]</td>
<td>11-3=8[Nm]</td>
</tr>
</tbody>
</table>

TABLE III

Sensitivity of the Actuator Faults Diagnosis Strategy.
Actuator 1

Actuator fault signal

Fault signal reconstruction

Fig. 5. FD experiment (Δτ, ̂Δτ signals). Sub-Optimal.

Actuator 2

Actuator fault signal

Fault signal reconstruction

Actuator 3

Actuator fault signal

Fault signal reconstruction

Fig. 6. Sensor 2 FD experiments: Sub-Optimal input law.

Fig. 7. Sensor 2 FD experiments: Super-Twisting input law.

E. Numerical Comparison of the Experimental Results

In order to compare the performance of the two observer approaches, the signals ̂Δτ(t) and ρ(j)(t) have been evaluated in case of actuators and sensors faults. Let t_f the time instant in which the fault occurs. The sensitivity of the proposed approach to a fault occurring on the actuator j, is evaluated as

\[ FS \left( \hat{\Delta} \tau_j(t) \right) = \max_{t > t_f} \left| \hat{\Delta} \tau_j(t) \right| - \max_{0 < t < t_f} \left| \hat{\Delta} \tau_j(t) \right| \]  \hspace{1cm} (22)

while the sensitivity of the proposed approach to a fault occurring on the sensor j, is evaluated as

\[ FS \left( \rho_j^{(i)}(t) \right) = \max_{t > t_f} \left| \rho_j^{(i)}(t) \right| - \max_{0 < t < t_f} \left| \rho_j^{(i)}(t) \right|. \]  \hspace{1cm} (23)

First experiment: \( \Delta q_1 = 8, \Delta q_2 = \Delta q_3 = 0(\text{deg}) \)

<table>
<thead>
<tr>
<th>Obs. (1)</th>
<th>Obs. (2)</th>
<th>Obs. (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( FS \left( \rho_1^{(1)}(t) \right) )</td>
<td>2.06-0.28=1.78</td>
<td>2.06-0.24=1.82</td>
</tr>
<tr>
<td>( FS \left( \rho_2^{(3)}(t) \right) )</td>
<td>2.40-0.72=1.68</td>
<td>0.33-0.54=0.21</td>
</tr>
<tr>
<td>( FS \left( \rho_3^{(1)}(t) \right) )</td>
<td>4.90-5.00=-0.10</td>
<td>4.54-5.18=-0.64</td>
</tr>
</tbody>
</table>

Second experiment: \( \Delta q_2 = -12, \Delta q_1 = \Delta q_3 = 0(\text{deg}) \)

<table>
<thead>
<tr>
<th>Obs. (1)</th>
<th>Obs. (2)</th>
<th>Obs. (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( FS \left( \rho_1^{(1)}(t) \right) )</td>
<td>0.92-0.32=0.60</td>
<td>0.90-0.28=0.62</td>
</tr>
<tr>
<td>( FS \left( \rho_2^{(3)}(t) \right) )</td>
<td>5.20-0.80=4.40</td>
<td>5.41-0.70=4.71</td>
</tr>
<tr>
<td>( FS \left( \rho_3^{(1)}(t) \right) )</td>
<td>8.50-4.06=4.44</td>
<td>7.54-4.30=3.24</td>
</tr>
</tbody>
</table>

Third experiment: \( \Delta q_2 = 11, \Delta q_1 = \Delta q_3 = 0(\text{deg}) \)

<table>
<thead>
<tr>
<th>Obs. (1)</th>
<th>Obs. (2)</th>
<th>Obs. (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( FS \left( \rho_1^{(1)}(t) \right) )</td>
<td>0.28-0.18=0.10</td>
<td>0.23-0.18=0.05</td>
</tr>
<tr>
<td>( FS \left( \rho_2^{(3)}(t) \right) )</td>
<td>0.88-0.64=0.24</td>
<td>0.78-0.77=0.01</td>
</tr>
<tr>
<td>( FS \left( \rho_3^{(1)}(t) \right) )</td>
<td>11.50-3.28=8.22</td>
<td>10.95-3.30=7.65</td>
</tr>
</tbody>
</table>

Table IV

Sensitivity of the sensor faults diagnosis strategy in [rad/s²]. Sub-Optimal algorithm is adopted.

Positive values of \( FS(\cdot) \) indicate that the fault has been detected by the proposed approach, while null or negative values of \( FS(\cdot) \) denote that the fault cannot be detected by relying on the proposed approach. According to the properties...
of the proposed modified GOS to detect sensor faults, a single fault occurring on the \( j \)th sensor can be correctly isolated only if it produces significant effects on the observers \((i), i \neq j, \) but not on the observer \((i), i = j. \) To evaluate this property, when a fault occurs on sensor \( i, \) the tolerance of the \((i)\)th GOS observer to fault occurring on sensor \( i, \) is given as

\[
FT_{ij} (\rho_j^{(i)} (t)) = \max_{t>t_f} |\rho_j^{(i)} (t)| \quad j \neq i. \tag{24}
\]

Lower values of \(FT_{ij}(\cdot)\) indicate better tolerance of the observer \((j)\) versus a faults occurring on sensor \(i.\) Tables III, IV, and V the values of \(FS(\Delta\tau_j(t))\) and \(FS(\rho_j^{(i)}(t))\) have been shown in different situations. Table III reports the values of \(FS(\Delta\tau_j(t))\) which have been evaluated in case of three non simultaneous actuator faults \(\Delta\tau(t),\) for each one of the input signals \(\tau(t).\) First column report the results obtained with the Sub-Optimal, the second column report the results obtained with Super-Twisting approach. In Tables IV, and V, the same evaluation is made in case of sensor faults. Three different experiments have been made. For each experiment, a single non simultaneous fault has been injected on a particular sensor. In Table IV are reported the values of \(FS(\rho_j^{(i)}(t))\) using Sub-Optimal, while Table V reports the same values obtained using Super-Twisting. The values of \(FT_{ij}(\rho_j^{(i)}(t))\) are also reported in these two tables (see the first value of each cell, in the column referred to the observer associated to the faulty sensor).

| First experiment: \(\Delta q_1 = 8, \Delta q_2 = \Delta q_3 = 0\) deg |
|------------------------|----------------|----------------|
| \(FS(\rho_1^{(1)}(t))\) | \(1.20 \pm 0.25 = 0.95\) | \(1.19 \pm 0.22 = 0.97\) |
| \(FS(\rho_2^{(1)}(t))\) | \(1.55 \pm 0.60 = 0.95\) | \(1.00 \pm 0.52 = 0.48\) |
| \(FS(\rho_3^{(1)}(t))\) | \(1.15 \pm 0.18 = 0.65\) | \(1.83 \pm 1.54 = 0.29\) |

| Second experiment: \(\Delta q_2 = -12, \Delta q_1 = \Delta q_3 = 0\) deg |
|------------------------|----------------|----------------|
| \(FS(\rho_1^{(1)}(t))\) | \(0.90 \pm 0.28 = 0.62\) | \(0.50 \pm 0.25 = 0.25\) |
| \(FS(\rho_2^{(1)}(t))\) | \(2.35 \pm 0.70 = 1.65\) | \(2.35 \pm 0.6 = 1.75\) |
| \(FS(\rho_3^{(1)}(t))\) | \(1.66 \pm 1.28 = 0.38\) | \(2.00 \pm 1.31 = 0.69\) |

| Third experiment: \(\Delta q_3 = 11, \Delta q_1 = \Delta q_2 = 0\) deg |
|------------------------|----------------|----------------|
| \(FS(\rho_1^{(1)}(t))\) | \(0.20 \pm 0.16 = 0.04\) | \(0.21 \pm 0.17 = 0.04\) |
| \(FS(\rho_2^{(1)}(t))\) | \(0.73 \pm 0.54 = 0.19\) | \(0.67 \pm 0.68 = 0.01\) |
| \(FS(\rho_3^{(1)}(t))\) | \(1.80 \pm 1.12 = 0.68\) | \(1.81 \pm 1.03 = 0.78\) |

TABLE V
Sensitivity of the sensor faults diagnosis [rad/s²]

Super-Twisting algorithm

From the numerical comparison shown in the tables, the Sub-Optimal Algorithm showed higher sensitivity values with respect to Super-Twisting Algorithm. In contrast better performances in terms of tolerance have been obtained by considering the Super-Twisting approach, as the values relative to \(FT_{ij}(\cdot)\) shown in Table V are smaller in all cases with respect to those obtained by applying the Sub-Optimal approach.

VII. CONCLUSIONS

A FD scheme for robot manipulators based on the concept of second order sliding modes has been presented in this paper. The scheme presented in this paper allows to deal with the following cases: faults (even multiple) occurring only on the actuators and single faults occurring on actuators or sensors. The detection of the faults presence on actuators is performed relying on Unknown Input Observers (UIOs), while the detection of the faults presence on sensors is performed relying on a modified Generalized Observers Scheme (GOS). The observers input laws are designed according to two SOSM methodologies: the Sub Optimal approach and the Super-Twisting approach. The proposed fault diagnosis approach can be extended also to some parametric faults (non-additive). Future work will be devoted to deal with this topic. Experimental results on a real COMAU SMART3-S2 are presented. The numerical comparison showed that the proposed approach, when Sub-Optimal is adopted for the input laws, has good sensitivity in the detection both for the actuators and for the sensors. In contrast, when Super-Twisting approach is adopted for the observers involved in the GOS, it is possible to obtain better tolerance properties.

REFERENCES

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